MTL-108: Major Exam on 29/04/2019

Max Time: 2 hrs Max Marks 40

Any form of wrong practice during exam will fetch you (-20) marks without any warning.

1. Let X and Y denote the values of two stocks at the end of a five-year period, where X is uniformly distributed on the interval $(0, 12)$. And given $X = x$, the random variable Y is uniformly distributed on the interval $(0, x)$. Find the correlation $\rho_{X,Y}$ of X and Y. [5]

2. A car is new at the beginning of a calendar year. The time, in years, before the car experiences its first failure is exponentially distributed with mean 4. Find the probability that the car experiences its first failure in the first quarter of some calendar year. Assume that the car has infinite life of usage. [5]

3. Let X be an $N(0,1)$ random variable, Y be a χ^2_n random variable, and X and Y are independent. Using the method of transformation, determine the pdf of the random variable $T = \dfrac{X}{\sqrt{\dfrac{Y}{n}}}$. [5]

4. A sample of 5 iid observations, $\{x_1 = 0.1,\ x_2 = 0.2,\ x_3 = 0.4,\ x_4 = 0.7,\ x_5 = 0.9\}$ is collected from a continuous distribution with pdf $f(x) = \theta x^{\theta-1}$, $0 < x < 1$. Estimate the maximum likelihood value of θ. Give details of your working. [5]

5. A tire manufacturer produces tires that last, on average, at least 25000 miles when the production process is working properly. Based on past experience, the standard deviation of the tires is found to be 3500 miles. The operations manager will stop the production process if there is evidence that the average tire life is below 25000 miles. A random sample of 100 tires is selected and the operations manager is willing to take a risk of committing a Type I error with probability $\alpha = 0.05$. Compute the power of the test $1 - \beta$ if the population average life is actually 23500 miles. [5]

6. A random sample $X_1, \ldots, X_8$ is taken from a normally distributed population $N(\mu, \sigma^2)$. We wish to test whether the variance σ^2 can be considered equal to 2, against the hypothesis that it is not equal to 2. It is given that sample mean $\bar{x} = 3$ and $\sum_{i=1}^{8}(x_i - 3)^2 = 3.25$. Test the hypothesis at both 2% and 5% levels of significance and draw appropriate conclusion from it. [5]

7. Suppose we are interested in finding a 95% confidence interval for the proportion of undergraduate students from the state of Delhi-NCR. We take a random sample of 200 students, and find that 170 of them are from Delhi-NCR. Construct the confidence interval for the proportion. [4]

8. The data, based on independent samples drawn from normal populations, to study the difference in the two averages. The data summary is as follows:

$$n_1 = 16,\ \ n_2 = 10,\ \ ,\bar{x}_1 = 0.94,\ \ \bar{x}_2 = 0.88,\ \ s_1^2 = 0.04,\ \ s_2^2 = 0.09$$

(a) Test the null hypothesis that the variances of the two population are equal at 10% level of significance.

(b) Use part (a), to test the difference between the two averages at 10% level of significance. [6]