MTL108: Introduction to Statistics
Minor-1 on February 07, 2018

Max Marks: 20 **Time: 1 Hr**

Q1. Find the probability that a person flipping a fair coin gets
(a) the third head on the seventh flip.
(b) the first head on the fourth flip. [2]

Q2. (a) In a certain machine, multiple engines operate independently and each can fail with the probability equal to 0.4. Suppose that the machines runs smoothly if at least one-half of its engines are in working conditions. Determine whether a 4-engine machine or a 2- engine machine has the higher probability for a smooth running. Give reason. [3]

Q3. Suppose a certain class of small companies have difficulty making a profit in their first year of operation. Let X be the proportion of this class that can make profit and let the probability density function of X be given by

$$f(x) = \begin{cases} kx^3(1-x)^2 & 0 \le x \le 1 \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability that at least 80% of the companies make a profit in the first year. [3]

Q4. Let the joint density function of two random variables X and Y be given by

$$f(x,y) = \begin{cases} \dfrac{x(1+3y^2)}{4} & 0 < x < 2,\ 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$$

Evaluate $P(\frac{1}{4} < Y < \frac{1}{2} \,|\, X = \frac{1}{3})$. Are X and Y independent random variables? Justify. [4]

Q5. A random variable X has a mean $\mu = 10$ and a variance $\sigma^2 = 4$. Use Chebyshev's theorem, to determine
(a) bound of $P(|X - 10| \ge 3)$
(b) bound of $P(5 < X < 15)$
(c) the value of the constant c such that $P(|X - 10| \ge c) \le 0.01$. [4]

Q6. Let X be a geometric random variable following the pdf

$$P(X = x) = pq^{x-1},\ x = 0, 1, 2, \ldots,$$

where $q = 1 - p,\ p \in (0, 1)$. Determine the moment generating function of X and use it to determine the second order moment of X about the origin. [4]