

Time allowed: 1 hour

1. Let X_1, \dots, X_5 be a random sample from $N(0, \sigma^2)$. Find a constant c such that $Y = \frac{c(X_1 - X_2)}{\sqrt{X_3^2 + X_4^2 + X_5^2}}$ has a t -distribution. Also, find $E(Y)$. (3 + 2 marks)

2. Suppose that 200 independent observations X_1, X_2, \dots, X_{200} are obtained from random variable X . We are told that $\sum_{i=1}^{200} X_i = 400$ and that $\sum_{i=1}^{200} X_i^2 = 4600$. Using these values obtain unbiased estimates for $E(X)$ and $Var(X)$. Also, find a biased estimates for $E(X)$. (2 + 2 + 1 marks)

3. A random variable X has pdf

$$f(x) = \frac{1}{2}e^{-|x-\theta|}, \quad -\infty < x < \infty.$$

Obtain the ML estimates of θ based on a random sample X_1, X_2, \dots, X_n . (5 marks)

4. To examine if two catalysts are equivalent in terms of the mean yield of a chemical process, $n_1 = 8$ chemical process are performed with catalyst A , and $n_2 = 8$ are performed with catalyst B . From catalyst A , we obtain the sample mean $\bar{X} = 92.255$, the sample standard deviation $S_1 = 2.39$. From catalyst B , we obtain the sample mean $\bar{Y} = 92.733$, the sample standard deviation $S_2 = 2.98$. Test $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$ at $\alpha = 0.05$. (5 marks)

5. Suppose the number of traffic accidents occurs throughout the week in Delhi - Gurgaon national highway is as follows:

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Observed	23	18	17	19	23

Test the hypothesis that the number of traffic accidents occurs uniformly throughout the week. Assume $\alpha = 0.05$. (5 marks)

Table Values

$$P(\text{Z is a standard normal distribution} > Z_\alpha) = \alpha$$

$$P(\text{t r.v. with } n \text{ degrees of freedom} > t_{n,\alpha}) = \alpha$$

$$P(\chi^2 \text{ r.v. with } n \text{ degrees of freedom} > \chi_{n,\alpha}^2) = \alpha$$

$$P(\text{F r.v. with } n_1 \text{ and } n_2 \text{ degrees of freedom} > F_{n_1, n_2, \alpha}) = \alpha$$

$$Z_{0.15} = 1.04; Z_{0.10} = 1.285; Z_{0.075} = 1.44; Z_{0.05} = 1.645; Z_{0.025} = 1.96; Z_{0.01} = 2.33$$

$$t_{8,0.025} = 2.31; t_{9,0.025} = 2.26; t_{10,0.025} = 2.22; t_{14,0.025} = 2.145$$

$$\chi_{9,0.05}^2 = 16.917; \chi_{6,0.05}^2 = 12.6; \chi_{5,0.05}^2 = 11.1; \chi_{4,0.05}^2 = 9.48; \chi_{3,0.05}^2 = 7.81; \chi_{2,0.05}^2 = 5.99$$

$$F_{9,11,0.025} = 0.1539; F_{10,15,0.025} = 3.5217; F_{15,10,0.025} = 3.0602$$

Note: If above table values are not matched with your answer, please leave the answer without numerical.