



Max Marks: 20

Time: 1 Hr

-25 marks will be awarded by invigilator, without warning, if found indulging in any wrong doing.  
Additional -5 marks will be awarded for clarifying reasons against wrong conduct at later time.

1. Let  $X$  and  $Y$  be independent random variables such that  $X \sim N(0, 1)$  and  $Y \sim N(0, 1)$ . Define two random variables  $U = \frac{X}{Y}$  and  $V = |Y|$ .

$e^{-t}$   $\int e^{-x} p(x)$

- (a) Find the joint distribution of  $U$  and  $V$ .
- (b) Using (a), determine the marginal distribution of  $U$ .

[3+2]

2. Determine the mean (expectation) of an  $F$ -distributed random variable with  $\nu_1$  and  $\nu_2$  degrees of freedom.

$E(f)$

[4]

3. Two kinds of thread are being compared for strength. Fifty pieces of each type A and B of thread are tested under similar conditions. Brand A had an average tensile strength of 78 kilograms with a standard deviation of 5 kilograms, while brand B had an average tensile strength of 87 kilograms with a standard deviation of 6 kilograms. Construct a 95% confidence interval for the difference of the population means.

[3]

4. A marketing group conduct a survey to find the proportion ( $p$ ) of consumers who bought the newest generation mobile happy with their purchase.

(a) How large a sample size should be taken to estimate  $p$  with 2% margin of error and 90% confidence? Use  $\hat{p} = 0.5$  as a conservative guess.

b) The marketing group took a random sample of 1000 consumers who recently purchased this mobile and found that 400 are happy with their purchase. Determine the 90% confidence interval for  $p$ .

[2+2]

5. State which of the following statements are TRUE and FALSE with appropriate reasoning. NO marks will be awarded if the answer is not supported by correct justification.

(a) The researchers are not satisfied with their current confidence interval and want to shorten its length. To achieve this, they must decrease their confidence level and increase their sample size.

$p = 0.65$

$\bar{X} - \left(\frac{s}{\sqrt{n}}\right) z < u$

(b) It is known that 65% of all IITians prefer tea over coffee. If a random sample of 1000 IITians is picked then the standard error in the sample proportion is 0.47697.

(c) The time between arrivals of what's-app messages at your mobile is exponentially distributed with a mean 1.5 minutes. The probability that you do not receive a message during a one minute is 0.5134.

(d) The expectation of the beta distributed random variable  $X \sim B_1(\alpha = 6, \beta = 2)$  is 0.25.

$(1 - 2t)^{-6}$

[4]