DEPARTMENT OF MATHEMATICS

MTL 122: Real and Complex Analysis

Major *Marks - 50*

Answer any TEN of the following. All questions carry equal marks.

(1) Discuss the analyticity of the function $f(z) = \begin{cases} e^{-(z^{(-4)})}, & z \neq 0; \\ 0, & z = 0. \end{cases}$

(2) Let G be a region and define $G^* = \{z : \overline{z} \in G\}$. If $f : G \to \mathbb{C}$ is analytic then prove that $f^* : G^* \to \mathbb{C}$, defined by $f^*(z) = \overline{f(\overline{z})}$ is also analytic.

(3) Let $g(z)$ be a non vanishing analytic function on $\overline{D}$ and

$$f(z) = (z-a)^n g(z)$$

where $a \in D$, $n \in \mathbb{Z}$. Evaluate $\frac{1}{2\pi i}\int_{\partial D} \frac{f'(z)}{f(z)} dz$.

$n(z-a)^{n-1}g(z)$

(4) Evaluate the integral $\int_C \log z dz$, where C is the unit circle $|z| =$

(5) Suppose that the simple closed contour C encloses $z = z_0$ in a positive sense and that f is an analytic function. Show that

$$\int_C (z-z_0)^n dz = \begin{cases} 2\pi i, & \text{if } n = -1; \\ 0, & \text{if n is any other integer.} \end{cases}$$

and

$$\int_C \frac{f'(z)dz}{z-z_0} = \int_C \frac{f(z)dz}{(z-z_0)^2}$$

(6) The real parts of these analytic functions are

$$\sin x \cosh y;\ e^{y^2-x^2}\cos 2xy$$

respectively. Find their complex conjugates.

(7) Let f be an entire function such that $Re(f(z)) > M$ for all $z \in \mathbb{C}$. Show that f is a constant function.

(8) Obtain Taylor's series expansion of $f(z) = \frac{1}{z^2+4}$ about the point $z = -i$. Describe the region of where this convergence is valid.

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