DEPARTMENT OF MATHEMATICS

MTL 122: Real and Complex Analysis

Minor - 1 *Marks - 25*

I. Answer any three of the following. All questions carry equal marks. [*15 Marks*]

(a) Prove that every bounded sequence in $\mathbb{R}$ has a convergent subsequence.

(b) Let A be a subset of a metric space (X, d). Show that $x \in \overline{A}$ if and only if $d(x, A) = \inf\{d(x, y) : y \in A\} = 0$.

(c) Prove that in a metric space (X, d), X and $\emptyset$ are the only two clopen(both closed and open) sets if and only if every continuous function $f : X \to \{0, 1\}$ is constant, where $\{0, 1\}$ is considered to be a discrete metric space.

(d) For two non-empty subsets A and B of $\mathbb{R}$, define $A + B = \{a + b : a \in A, b \in B\}$. Show that if A and B are compact, then $A + B$ is compact. Give an example where A and B are closed but A + B is not.

II. State whether the following are True or False. All questions carry equal marks. Justify your assertion. [*10 Marks*]

[no marks will be awarded if justification is not given]

(1) Let $X = \{a, b, c, d, e\}$ with the discrete metric. Then $A = \{a, c, d, e\}$ is dense in X.

(2) Consider the space $\mathbb{N}$, the set of natural numbers, as a subspace of $\mathbb{R}$. Then $\mathbb{N}$ is a Baire space.

(3) Let (X, d) be a metric space. For $A \subset X$, define $\chi_A : X \to \{0, 1\}$ as

$$\chi_A(x) = \begin{cases} 1 & x \in A, \\ 0 & \text{otherwise.} \end{cases}$$

Then χ_A is continuous iff A is both open and closed.

(4) The sequence $\{\sqrt[n]{e}\}$, where $e = 2.17\ldots$ is convergent.

(5) $\mathbb{Q}$ is homeomorphic to $\mathbb{N}$, both considered as subspaces of $\mathbb{R}$.