

Minor 2(Open book)

Aug. 2020

Real and Complex Analysis

MTL122/ MTL503/ MTL506

Total Marks: 20

Time: 1 day

- (1) If $f(z)$ is analytic for $|z| < 1$ and

$$|f(z)| \leq \frac{1}{1 - |z|},$$

find the best estimate of $f^n(0)$, using Cauchy's formula. **[5 marks]**

- (2) Let $w(z)$ be an analytic function in \mathbb{C} . Assume it has a non-essential singularity (pole of some order or removable) at ∞ . Then show that $w(z)$ is a polynomial. **[5 marks]**

- (3) Let $h(z) = \sum_{n=0}^{\infty} a_n z^n$, with positive radius of convergence. Is it possible for h to satisfy

$$h\left(\frac{1}{n^3}\right) = \frac{1}{n^6}$$

and

$$h\left(\frac{1}{n^2}\right) = \frac{1}{n^6}$$

for all $n \in \mathbb{N}$? If this holds, then what will be h ? **[6 marks]**

- (4) Find all the entire functions g that satisfy

$$g''\left(\frac{1}{n}\right) + g\left(\frac{1}{n}\right) = 0$$

for all $n \in \mathbb{N}$. **[4 marks]**