

MTL146: Combinatorics

Test (2021)

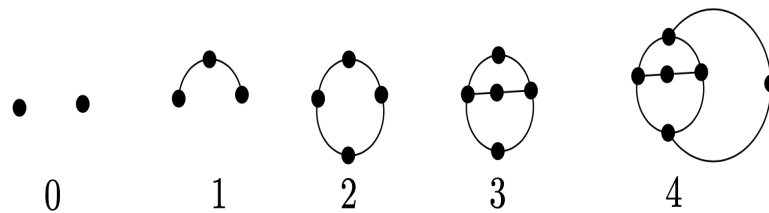
Time: 45 Min, Total marks: 15.

Instructions:

- Write your answers neatly and to the point.
- Remember that you will be graded on what you write and not what you intend to write.

Questions:

Q1 Consider the following game. Initially, n dots are drawn on a sheet of paper. The players alternate their moves, and the player with no legal move left loses. In each move, a player connects two dots with an arc and draws a new dot somewhere on the newly drawn arc. A dot can be used as an endpoint of a new arc only if there are at most 2 other arc ends leading into that dot, and a new arc must not cross any other arcs already drawn. An example of the game when $n = 2$:



Prove that a game with n initial dots lasts no more than $3n - 1$ moves (for any strategy of the players). (8)

Q2 For natural numbers $m \leq n$, we define a Latin $m \times n$ rectangle as a rectangular table with m rows and n columns with entries chosen from the set $\{1, 2, \dots, n\}$ and such that no row or column contains the same number twice. Count the number of all possible Latin $2 \times n$ rectangles. (7)