

MTL146: Combinatorics

Test (2021)

Time: 75 Min, Total marks: 35.

Instructions:

- Write your answers neatly and to the point.
 - Remember that you will be graded on what you write and not what you intend to write.
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Questions:

Q1 Suppose that there are s MOLS of order n . Prove that there are $s - 1$ MOLS of order n , all of which are idempotent. (13)

Q2 Suppose that (X, \mathcal{A}) is a $(v, 3, 1)$ -BIBD and (X, o) is any quasigroup of order v . Define $Y = X \times \{1, 2, 3\}$. For $1 \leq i \leq 3$ and for any $A \in \mathcal{A}$, define

$$A_i = \{(x, i) : x \in A\}.$$

Define

$$\mathcal{B}_1 = \{A_i : 1 \leq i \leq 3\}$$

and define

$$\mathcal{B}_2 = \{(x, 1), (y, 2), (x \circ y, 3) : x, y \in X\}.$$

Prove that $(Y, \mathcal{B}_1 \cup \mathcal{B}_2)$ is a $(3v, 3, 1)$ -BIBD. (11)

Q3 Suppose (V, \mathcal{B}) is a t - (v, k, λ) design. Let z be a positive integer such that $z \leq t$. Prove or disprove that each z -element subset of the set V is contained in same number of blocks of \mathcal{B} . (11)