Major Examination
MTL 180: Discrete Mathematical Structures
19th November 2015

Total Marks: 40 | Time: 2 hours

Please give adequate explanation for full credit.

1. Consider the hypotheses "If you send me an e-mail message, then I will finish writing the program", "If you do not send me an e-mail message, then I will go to sleep early", and "If I go to sleep early, then I will wake up feeling refreshed". Symbolize these hypotheses and use rules of inference to arrive at the conclusion "If I do not finish writing the program, then I will wake up feeling refreshed". [3]

2. Let X be a partially ordered set. Show that one can write X as a union of two disjoint sets A and B such that A is well ordered (with respect to the ordering in X) and B has no least element. [4]

3. Use a generating function to show that every positive integer can be uniquely expressed as a sum of *distinct* powers of 2. [5]

4. Using generating function, evaluate the sum $\sum_{k=1}^{n} k \cdot 3^k \binom{n}{k}$. [4]

5. (a) The cube graph Q_n is defined as follows: the vertices of Q_n are all sequences of length n with entries from $\{0, 1\}$ and two sequences are joined by an edge if they differ in exactly one position. How many edges does Q_n have? Which cube graphs Q_n have an Euler circuit? [3+1]

   (b) Let G be a k-regular graph, and let the length of the shortest cycle of G be 4. Prove that G has at least $2k$ vertices. [3]

6. (a) Show that if G is a simple connected bipartite planar graph with n ($n \geq 3$) vertices and m edges, then $m \leq 2n - 4$. [3]

   (b) Let G be a simple planar connected 3-regular graph. Prove that $\sum_{i \geq 3} (6 - i) f_i = 12$, where f_i is the number of faces of G each of which is bounded by i edges. [4]

   (c) Let G be a simple graph with chromatic number $\chi(G) = 31$. Prove that G has at least 465 edges. [3]

7. (a) For any prime $p \geq 5$, prove that [3]

$$\sum_{k=1}^{p-1} \frac{(p-1)!}{k} \equiv 0 \pmod{p^2}.$$

   (b) Let $p \geq 5$ be a prime, and write [4]

$$1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{p} = \frac{r}{ps}.$$

   Prove that $r \equiv s \pmod{p^3}$.

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n-m+f=2
8: 2+
2f= 4+2