

Minor-II

MTL 180: Discrete Mathematical Structures

Total Marks: 20

8th October 2015

Time: 1 hour

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1. (a) Find the order of 2 modulo 101. [3]
(b) Using Quadratic Reciprocity Law, check if the equation $x^2 \equiv 94 \pmod{257}$ has any solution. [3]
2. Let $n \geq 2$. Let $p = 2^n + 1$ be a prime.
(a) Prove that n is even. [1]
(b) Prove that $\left(\frac{3}{p}\right) = -1$. [3]
(c) Deduce that $3^{(p-1)/2} + 1$ is divisible by p . [2]
3. For a nonempty $X \subseteq \{1, 2, 3, \dots, 14, 15\}$, define $\sigma(X) = \sum_{x \in X} x$, the sum of the elements of X . Using Pigeonhole principle, prove that among *any* 43 nonempty subsets of $\{1, 2, 3, \dots, 14, 15\}$, each having size at most three, there are subsets A and B such that $\sigma(A) = \sigma(B)$. [5]
4. Establish the following identity by a *combinatorial argument*: [3]

$$\binom{3n}{3} = 3 \cdot \binom{n}{3} + 6n \cdot \binom{n}{2} + n^3.$$
