Department of Mathematics
MTL 390/MAL 390 (Statistical Methods)
Major Test (II Semester 2015 - 2016)

Time allowed: 2 hours **Max. Marks: 40**

1. Suppose that population distribution is an uniform distribution on the interval $(0, 1)$ and $X_1, X_2, \ldots, X_n$ is a random sample from this population. Find the sampling distribution of

$$\left(\prod_{i=1}^{n} X_i\right)^{1/n}.$$

(4 marks)

2. An economist estimates that the average Indian household saves 15% of its income. In a random sample of 64 households, the average saving is found to be 14%, and the standard deviation is 7%. Assume that population is normally distributed. Do we have enough evidence to refute the economist's claim? Use a 0.05 level of significance level. (4 marks)

3. Suppose that a public opinion poll is surveyed with random sample of 1000 voters for 16th Lok Sabha Election 2014. Respondents were classified by gender (male or female) and by voting preference (Congress, BJP or CPI-M). Results are shown in the contingency table below.

	Voting Preference		
	Congress	BJP	CPI-M
Male	200	150	50
Female	250	300	50

Can it be concluded from the above sample data that there is no relationship between gender and voting preference. Use a 0.05 level of significance level. (4 marks)

4. Merchant vessels of a certain type were exposed to risk of accident through heavy weather, ice, fire, grounding, breakdown of machinery, etc for a period of 400 days. The number of accidents to each vessel, say X, may be considered as a random variable. The following data were reported.

Number of accidents (X)	0	1	2	3	4	5	6
Number of vessels with X *accidents*	1448	805	206	34	4	2	1

Test the hypothesis that the number of accidents during the specified 400 day period have a Poisson distribution. Assume $\alpha = 0.05$. (4 marks)

5. Consider the 12 pairs of observations of (X, Y) are as follows:

X	50	11	2	19	26	73	81	51	11	2	19	25
Y	22.1	35.9	57.9	22.2	42.4	5.8	3.6	21.4	55.2	33.3	32.4	38.4

Test the hypothesis $H_0 : \rho = 0$ against $H_1 : \rho \neq 0$ where ρ is the population correction coefficient with 0.05 level of significance level. (6 marks)

6. (a) Describe simple linear regression model.

   (b) Derive the estimators for the intercept and slope parameters using least square method. Further, prove that, these estimates are unbiased also.

(2 + 3 + 1 marks)

7. Consider the situation where we tabulate as follows the numbers of errors made by a group of 10 subjects in translating two passages of English, of equal length, into Spanish.

Subject No.	1	2	3	4	5	6	7	8	9	10
Errors in passage A	8	7	4	2	4	10	17	3	2	11
Errors in passage B	10	6	4	5	7	11	15	6	3	14

Using Wilcoxon rank sum test, test whether there is any significant difference between the two sets of scores, at 0.05 level of significance level. (5 marks)

8. (a) Describe AR(2) model.

   (b) Discuss the relationship between AR(1) model and simple linear regression model.

   (c) Find the confidence interval for the autoregressive parameter of AR(1) model with 95% confidence interval with sample of 300 data. Assume that, the estimated AR(1) coefficient is 0.60.

(2 + 2 + 3 marks)

Table Values

P(**Z** is a standard normal distribution $\geq Z_\alpha$) $= \alpha$

P(χ^2 r.v. with n degrees of freedom $\geq \chi^2_{n,\alpha}$) $= \alpha$

P(**t** r.v. with n degrees of freedom $\geq t_{n,\alpha}$) $= \alpha$

P(**F** r.v. with n_1 and n_2 degrees of freedom $\geq F_{n_1,n_2,\alpha}$) $= \alpha$

$Z_{0.025} = 1.96$; $Z_{0.05} = 1.645$; $Z_{0.0764} = 1.43$; $Z_{0.01} = 2.33$; $Z_{0.035} = 1.81$

$\chi^2_{2,0.05} = 5.99$; $\chi^2_{3,0.05} = 7.81$; $\chi^2_{4,0.05} = 9.48$; $\chi^2_{5,0.05} = 11.1$; $\chi^2_{6,0.05} = 12.6$

$t_{8,0.025} = 2.31$; $t_{9,0.025} = 2.26$; $t_{10,0.025} = 2.22$

$F_{9,11,0.025} = 0.1539$; $F_{10,15,0.025} = 3.5217$; $F_{15,10,0.025} = 3.0602$

Note: If above table values are not matched, please leave the answer without numerical.