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MTL390: STATISTICAL METHODS
MAJOR EXAMINATION

Date: 02/05/2023 **Writing Time: 1 hr 40 min** **Maximum Marks: 25**

1. a) Use **Kolmogorov Smirnov One Sample test** for testing at 1% and 5% level of significance the null hypothesis,

H_0: the sample values are chosen from $N(\mu, \sigma^2)$ with $\mu = 200$ and $\sigma^2 = 100$

Vs

H_1: the sample is from some other distribution

When the sample values are 184.7, 173.9, 185.3, 187.4, 215.9, 216.8

Use the following table to get the corresponding critical regions.

Kolmogorov-Smirnov One-Sided Test

n	0.1	0.05	0.025	0.01	0.005
1	0.9000	0.9500	0.9750	0.9900	0.9950
2	0.6838	0.7764	0.8419	0.9000	0.9293
3	0.5648	0.6360	0.7076	0.7846	0.8290
4	0.4927	0.5652	0.6239	0.6889	0.7342
5	0.4470	0.5094	0.5633	0.6272	0.6685
6	0.4104	0.4680	0.5193	0.5774	0.6166
7	0.3815	0.4361	0.4834	0.5384	0.5758
8	0.3583	0.4096	0.4543	0.5065	0.5418
9	0.3391	0.3875	0.4300	0.4796	0.5133
10	0.3226	0.3687	0.4092	0.4566	0.4889

[6]

b) Consider two independent samples (X and Y) of sizes $m = 14$ and $n = 16$ respectively. Let m_1 and m_2 are population medians of X and Y respectively. Suppose the sum of ranks of the X and Y observations in the combined sample are $R_X = 275$ and $R_Y = 190$, respectively.

Use the **Wilcoxon Rank Sum Test** to test the null hypothesis, with Normal approximation without continuity correction

$$H_0: m_1 = m_2$$

Vs

$$H_1: m_1 > m_2$$

Test the null hypothesis against the alternative at 1% and 5% level of significance. **[3]**

2. a) Suppose 300 random samples of bulbs, each of sample size 7, are selected from an electrical equipment factory, and the number of defective bulbs in a sample is counted. The following data gives the frequency of number of defective bulbs in a sample for different values, 0, 1, 2, and 3+ (i.e. 3 or more).

Number of defective bulbs in a sample	0	1	2	3 or more
Frequency	166	94	32	8

Use Chi Square Goodness of Fit test, for testing the null hypothesis at 5% and 1% level of significance,

H_0 : the number of defective bulbs in each batch follows Poisson (0.6)

Vs

H_1: the sample is from some other distribution

Use the values of expected probabilities, and summands of the test statistic rounded up to five decimal places. Use the following table to get the corresponding critical regions.

[5]

The areas given across the top are the areas to the right of the critical value. To look up an area on the left, subtract it from one, and then look it up (ie: 0.05 on the left is 0.95 on the right)

df	0.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	---	---	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750

b) Let $X_1, X_2, \cdots, X_n$ be a random sample from Poisson distribution with parameter θ.

Find an MVUE for $\gamma(\theta) = \frac{e^{-\theta}\theta^4}{24}$

[3]

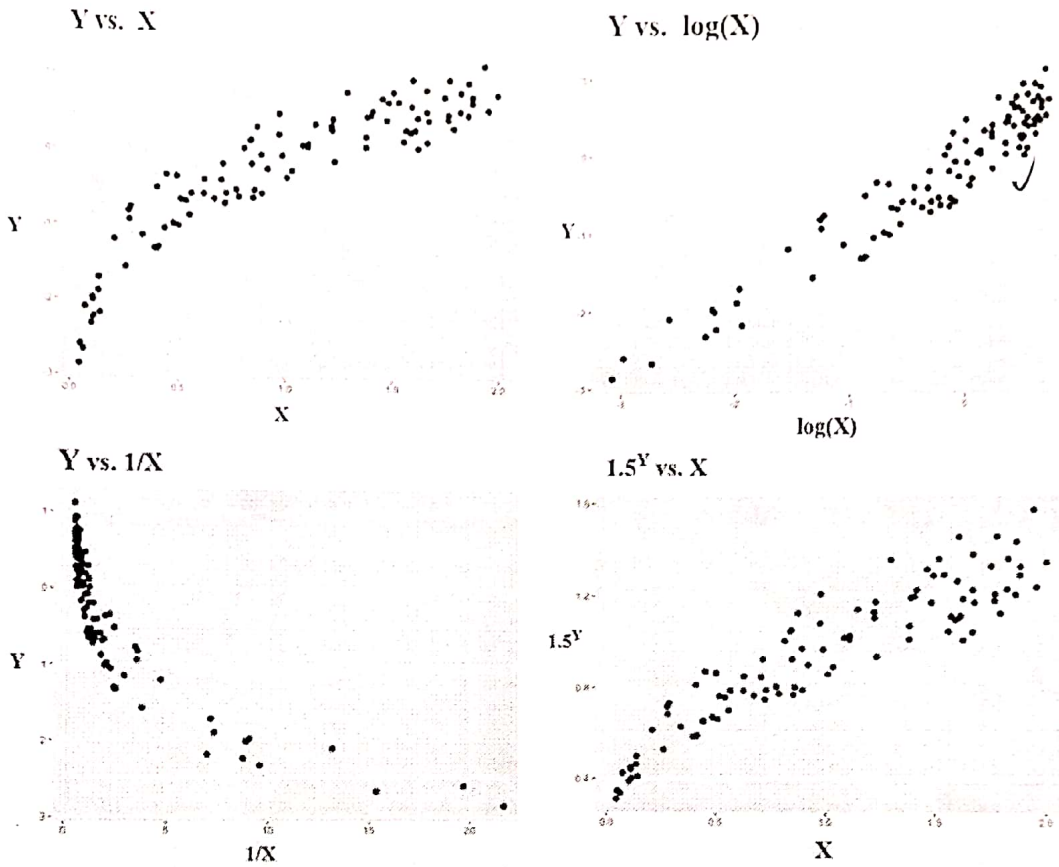
3. a) Let (X,Y) have the joint pdf given by,

$$f(x,y) = \begin{cases} 1, if\ |y| < x, 0 < x < 1 \\ 0, otherwise \end{cases}$$

Prove or disprove: (i) regression of Y on X is linear
(ii) regression of X on Y is not linear

[2 + 2 = 4]

b) Suppose we draw a random sample of size 100 from a bivariate population (X, Y) and we obtain the following sample plots.



Depending on these plots choose the most suitable transformation of data (if required) which indicates the **strongest linear relationship** to fit a regression model *M*
Consider a sample of size 12 from this population.

X	Y	log X	1/X	1.5^Y
0.53	-0.45	-0.63	1.89	0.83
0.79	-0.12	-0.23	1.26	0.95
0.52	-0.61	-0.65	1.92	0.78
1.27	-0.09	0.24	0.78	0.96
0.19	-1.35	-1.66	5.25	0.58
0.95	-0.29	-0.06	1.06	0.89
0.56	-0.74	-0.58	1.77	0.74
0.56	-0.28	-0.58	1.78	0.89
0.07	-2.16	-2.61	13.55	0.42
1.36	-0.11	0.3	0.74	0.96
1.71	0.08	0.54	0.58	1.03
1.64	0.16	0.49	0.61	1.07

i) Choose the plot corresponding to model *M* **[0.5]**
ii) Calculate the slope and intercept for model *M* **[3.5]**