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MTL390: Statistical Methods
Minor 2 Examination
Date: March 23, 2023 Time 13:00 – 14:00 Hours

1. Given one observation from a population with p.d.f. as

$$f(x,\theta) = \frac{2}{\theta^2}(\theta - x), 0 < x < \theta$$

Obtain the 98% confidence intervals for 0.

[4]

2. Let X be a random variable such that

$$P(X = x) = \begin{cases} \frac{\theta}{4N} + \frac{1}{2}\left(1 - \frac{\theta}{N}\right) & \text{for } x = 0 \\ \frac{\theta}{2N} + \frac{\alpha}{2}\left(1 - \frac{\theta}{N}\right) & \text{for } x = 1 \\ \frac{\theta}{4N} + \frac{1-\alpha}{2}\left(1 - \frac{\theta}{N}\right) & \text{for } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

Where N is known and θ, α are unknown.

If 90 independent observations on X yielded the values 0, 1, 2 with frequencies 28, 42, 20, respectively, Estimate α by **Method of Moments.**

[3.5]

3. Two random samples drawn from two normal populations are:

Sample 1	20	16	26	27	23	22	18	24	25	19		
Sample 2	27	33	42	35	32	34	38	28	41	43	30	37

Test at 5% and 1% level of significance whether the two populations have the same variances.

[3.5]

4. Consider the normal distribution $N(0,\theta)$. With a random sample $X_1, X_2 \cdots, X_n$. We want to estimate the standard deviation $\sqrt{\theta}$.

a) Find the constant c so that $T = c\sum_{i=1}^{n}|X_i|$ is an unbiased estimator of $\sqrt{\theta}$

[2]

b) Find the efficiency of the estimator T using CR bound.

[2]

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Quiz 4

1. a) Two populations have their means equal but standard deviation (S.D.) of one is twice that of the other. Samples of equal size of 2000 are taken from both populations. Show that the difference of means will not exceed 0.15σ, where σ is the smaller S.D. **[3]**

b) Let X ~ N $(\mu, 9)$. To test null $H_0: \mu = -1$ against $H_1: \mu = 1$, based on a sample of size 5 from the given population. Let the critical region be $x_1 + 3x_2 + 5x_3 + 7x_4 + 9x_5 \geq 0$. Find the power of the test. **[2]**

2. Let x_1, x_2 be two observations from a population with probability density function given by $f(x) = \theta x^{\theta-1}$ for $0 < x < 1$. Find the test with Most Powerful Critical Region corresponding to significance level $\alpha = \frac{15-ln(16)}{16}$, for testing the null hypothesis $H_0: \theta = \theta_0 = 4$ against $H_1: \theta = \theta_1$ for $\theta_1 > 4$. Let $\beta(\theta_1)$ be the power of above test. Calculate $\beta(5)$. **[5]**

3. An educator claims to have developed a training programme that can improve the score of the students of grade 10 (maximum score is 100). The underlying population F_X is continuous. To investigate this claim some researchers randomly chose 18 students from grade 10 and recorded their scores before and after the training programme. Does the data support the claim of the educator? (You may use Normal Approximation)

Use the paired sample sign test with normal approximation at 1% and 5% level of significance

Subject Number	Score befc	Score afte		Subject Number	Score bef	Score afte
1	51.50	59.0		10	72.50	75.00
2	68.50	77.00		11	88.00	89.50
3	79.50	81.50		12	62.75	73.50
4	87.00	92.25		13	48.50	54.75
5	58.50	51.00		14	45.00	49.50
6	98.00	98.00		15	77.25	75.25
7	73.50	78.50		16	93.00	94.50
8	56.50	54.00		17	82.75	85.00
9	67.00	64.50		18	78.50	73.50

[5]