INDIAN INSTITUTE OF TECHNOLOGY DELHI
DEPARTMENT OF MATHEMATICS
MTL 411 (FUNCTIONAL ANALYSIS)
Minor Examination

DATE: 15/03/2021 Total Marks: 30 Time: 3.30 – 5:00 pm

MARKS WILL BE AWARDED ONLY FOR THOSE ANSWERS WITH PROPER JUSTIFICATION

Question 1: Let X and Y be two Banach spaces and let $\{T_n\}$ be a sequence in $\mathcal{B}(X,Y)$. Assuming that for every $x \in X$, $T_n(x)$ converges as $n \to \infty$ to a limit denoted by $T(x)$. Show that if $x_n \to x$ in X, then

$$\lim_{n\to\infty} T_n(x_n) \to T(x) \quad \text{in} \quad Y.$$

[5]

Question 2: Let X and Y be Banach spaces. The set $\mathcal{A}$ of invertible transformations in $\mathcal{B}(X,Y)$ is open. [5]

Question 3: Suppose that X is a Banach space, Y is a normed space and $T \in \mathcal{B}(X,Y)$. If there exists $\alpha > 0$ such that

$$||T(x)|| \geq \alpha||x||, \quad \forall\, x \in X.$$

Then Image(T), is closed.

[5]

Question 4: Let $\{\alpha_n\}$ be a sequence in $\mathbb{R}$. Show that if $\sum_{n=1}^{\infty} \alpha_n\, x_n$ is finite for all $x = \{x_n\} \in l^1$, then $\{\alpha_n\} \in l^\infty$. [5]

Question 5: Let $T : l^2 \to l^2$ be the linear transformation defined by

$$T(x_1, x_2, x_3, x_4, \ldots) = (0, 4x_1, x_2, 4x_3, x_4, \ldots).$$

(a) Find the norm of T.

(b) Find T^2 and hence find $||T^2||$ and compare this with $||T||^2$.

[5]

Question 6 : Let X be a Banach space with norm $||.||_1$ and let Y be a Banach space with norm $||.||_2$. If $Z = X \times Y$ with

$$||(x,y)||_Z := ||x||_1 + ||y||_2.$$

Show that $(Z, ||(.,.)||_Z)$ is a Banach space. [5]

——ALL THE BEST——