

Department of Mathematics
Indian Institute of Technology Delhi
MTL 503 Real Analysis: Minor-1

Total marks: 20

Time: 1 hour

1. No marks will be provided if appropriate justification is not provided.
2. Every question is compulsory and carries equal weightage.

1. Let $f : X \rightarrow Y$ and let A and B be subsets of X and Y respectively. Show that

Ⓛ) $f(A \cap f^{-1}(B)) = f(A) \cap B.$

2. Show that \mathbb{R} and the Cantor set are numerically equivalent.

3. Show that the set

$$\{r \in \mathbb{Q} : r > 0 \text{ and } r^2 > 3\}$$

is a Dedekind cut.

Ⓛ) 4. State and prove a characterization for the limit inferior of a real sequence.

5. Let X be a finite set and let $Y = \mathcal{P}(X)$, the power set of X . Define $d : Y \times Y \rightarrow \mathbb{R}$

Ⓛ) by

$$d(A, B) = \#(A) + \#(B) - 2\#(A \cap B).$$

where $\#(A)$ denotes the cardinality of the subset A . Show that d is a metric on Y .

One MTE

$$0 < x$$

