MTL712 (Computational methods for differential eq.) IITD MAJOR EXAM

Duration of Examination: 2 hour November 2016

Instructions

1. The total number of marks are 50 (marks are indicated in the margin).

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1. Find the condition on step size h if two stage second order RK method applied to the system $\mathbf{y}' = -A\mathbf{y}$, where

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 6 \end{bmatrix},$$

is absolutely stable. What is the corresponding result for the matrix

$$A = \begin{bmatrix} 20 & 0 \\ 0 & 60 \end{bmatrix}.$$

[7]

2. Consider a k-step LMM

$$y_{n+k} + \alpha_{k-1}y_{n+k-1} + \alpha_{k-2}y_{n+k-2} + \ldots + \alpha_0 y_n = h\left(\beta_k f_{n+k} + \beta_{k-1}f_{n+k-1} + \ldots + \beta_0 f_n\right)$$

is used to solve the following IVP

$$y'(x) = f(x,y), \quad y(x_0) = y_0,$$

where $f(x,y)$ is non-linear. Compare the fixed point iteration method, predictor corrector method and Newton-Raphson method for solving the above problem. [7]

3. Prove that the difference scheme

$$u^{n+1} = Qu^n,$$

is stable with respect to the $\ell_{2,\Delta x}$ norm if and only if there exist positive constants Δt_0 and Δx_0 and non-negative constants β and K so that

$$|\rho(\xi)|^{n+1} \le Ke^{\beta(n+1)\Delta t},$$

for $0 < \Delta t \le \Delta t_0$, $0 < \Delta x \le \Delta x_0$ and all $\xi \in [-\pi, \pi]$. [7]

4. Consider the partial differential equation (PDE)

$$v_t = v_{xx} - v, t > 0, x \in (-\infty, \infty),$$

Analyze the stability of Crank-Nicolson scheme for the above PDE. [7]

k-1 k

5. Discuss the consistency of the difference scheme

$$u_k^{n+1} = (1-2r)u_k^n + r(u_{k+1}^n + u_{k-1}^n), k = 1, 2, ..., M-1; r = \frac{\nu \Delta t}{\Delta x^2},$$

$$u_M^{n+1} = 0,$$

$$u_0^n = u_1^n,$$

to the initial-boundary value problem

$$v_t = \nu v_{xx}, x \in (0,1), t > 0,$$

$$v(x,0) = f(x), x \in [0,1],$$

$$v(1,t) = 0, t > 0,$$

$$v_x(0,t) = 0, t > 0.$$

[7]

6. Consider the IVP

$$v_t + a v_x = \nu v_{xx}, v(x,0) = f(x).$$

(i). Analyze the stability of FTCS scheme for above IVP given that $r^2 \geq \frac{R^2}{4}$, where $r = \frac{\nu \Delta t}{\Delta x^2}$ and $R = \frac{a \Delta t}{\Delta x}$. [5]

(ii). Analyze the stability of the FTBS scheme for above IVP, when $\nu = 0$ and $a > 0$. [5]

(iii). Analyze the stability of FTCS scheme for above IVP, when $\nu = 0$ and $a > 0$. [4]

(iv). From above result (iii), what can you say about the stability of FTCS scheme for the following IBVP

$$v_t + a v_x = 0, v(x,0) = f(x), a > 0,$$

$$v(0,t) = v(1,t) = 0.$$

[1]

Give reason to support your answer.

THE END