MTL712 (Computational methods for differential eq.) IITD MINOR 1 EXAM

Duration of Examination: 1 hour

August 2016

Instructions

1. The total number of points is 25 (points are indicated in the margin).

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1. Prove that: Forward Euler method applied to the initial value problem

$$y' = f(x, y), \quad y(x_0) = y_0,$$

converges and the global error (e_n) is $O(h)$. It is given that $f(x, y)$ is continuous and $|f_y(x, y)| \leq K$ for some constant K. [8]

2. Consider a family of linear multi-step methods

$$y_{n+2} - 2ay_{n+1} + (2a - 1)y_n = h\left[af_{n+2} + (2 - 3a)f_{n+1}\right]$$

where a is a parameter.
(i). Under what conditions is the method consistent?
(ii). What is the maximum order of the method? What is the error constant in this case?
(iii). Under what conditions is it zero-stable?
(iv). Under what conditions is the method convergent? [8]

3. For the following initial value problem

$$y' = 2x + 3y, y(0) = 1.$$

(i). Use Taylor's series second order method to get $y(0.2)$ with step length $h = 0.1$ and $h = 0.2$ respectively.
(ii). Use analytic solution to calculate the error in each case.
(iii). Discuss the accuracy and efficiency of the method for $h = 0.1$ and $h = 0.2$. [6]

4. Find the region of absolute stability of the trapezoidal rule for the following initial value problem

$$y' = -y, \quad y(0) = y_0.$$

Is the method A_0 stable? [3]

THE END