MTL 712: MINOR EXAM

30 Marks total

Instructions: Write down all the steps of the solution clearly.

Problem 1 [6 marks] Find the truncation error of the Heun's method of order two used to solve the IVP $y'(x) = f(x, y), x \in (0, 1]$ with the initial condition $y(0) = 1$, considering $f(x, y)$ as a given function.

Problem 2 [6 marks] Consider the numerical method

$$y_{n+2} = 4y_{n+1} - 3y_n - 2hf(x_n, y_n).$$

$y' = f(x, y)$

(i) Is this method consistent? If yes, what is the order of consistency? Also, find the error constant.

(ii) Check whether or not this method is zero stable.

Problem 3 [7 marks] Assume that $u(x)$ is a positive, continuous function and that $\rho \geq 0,\ \epsilon \geq 0$. Then prove that the integral inequality

$$u(x) \leq \rho + \epsilon(x - x_0) + L \int_{x_0}^{x} u(t)\, dt,$$

implies the estimate

$$u(x) \leq \rho e^{L(x-x_0)} + \frac{\epsilon}{L}(e^{L(x-x_0)} - 1).$$

Problem 4 [6 marks] Which of the following methods are convergent? Provide proper justifications of your answer.

(i) $y_{n+3} - 2y_{n+2} + \frac{5}{4}y_{n+1} - \frac{1}{4}y_n = \frac{1}{4}hf(x_n, y_n).$

(ii) $y_{n+2} - y_{n+1} + 2y_n = -hf(x_n, y_n).$

(iii) $y_{n+4} = y_n + \frac{4h}{3}[2f(x_{n+3}, y_{n+3}) - f(x_{n+2}, y_{n+2}) + f(x_{n+1}, y_{n+1})].$

Problem 5 [5 marks] What values should the parameters a and b have so that the method:

$$y_{n+2} + y_{n+1} + ay_n = h(f(x_{n+2}, y_{n+2}) + bf(x_n, y_n)),$$

is consistent.