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MTL717: Major Exam on 06/05/2023

Max Marks: 35 Max Time: 2 hour

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1. Let X be a finite set and m be a basic probability assignment on the power set $P(X)$. Let F be a set of focal sets in $P(X)$ for m. Prove that if the finite body of evidence (F, m) is nested, then for $A, B, C \in P(X)$,

$$Bel(A \cup B \cup C) = Bel(A) + Bel(B) + Bel(C).$$

[4]

2. Suppose three basic probability assignments m_1, m_2, m_3 from three independent experts on a finite $P(X)$ are given as follows.

set	m_1	m_2	m_3
A	0.1	0.3	0.2
B	0.3	0.4	0.3
$A \cup B$	0.6	0.3	0.5

For every other subset $D \in P(X)$, we set $m(D) = 0$ and $m(\phi) = 0$.

Using the Dempster-Shafer evidence theory, determine the joint basic probability assignment $m_{(12)3}$. Tabulate the joint belief and plausibility measures corresponding to $m_{(12)3}$. [7]

3. For some $\lambda > -1$, let $g_\lambda : P(X) \to [0, 1]$ satisfy $g_\lambda(X) = 1$ and

$$g_\lambda(A \cup B) = g_\lambda(A) + g_\lambda(B) + \lambda g_\lambda(A) g_\lambda(B), \quad \text{if} \quad A \cap B = \emptyset.$$

(i) Show that g_λ is a fuzzy measure.

(ii) Let $X = \{x_1, \ldots, x_n\}$ and $A_i = \{x_1, \ldots, x_i\}$. Let $g_\lambda(\{x_i\}) = g_i$.

Prove that

$$\lambda + 1 = \prod_{i=1}^{n} (1 + \lambda g_i).$$

(iii) Take an example of $X = \{a, b, c\}$ and $g_\lambda(\{a\}) = 0.4$, $_\lambda(\{b\}) = 0.3$, $_\lambda(\{c\}) = 0.2$, find a non-zero λ and using the definition of $g_\lambda(\cdot)$, find the value of $g_\lambda(\{a, b\})$. [8]

4. For a fuzzy logic controller, we have the following rules.

Rule 1: If x is A_1 and y is B_1 then z is C_1
Rule 2: If x is A_2 and y is B_2 then z is C_2

Here, $A_1 = (-3, 0, 3)$, $B_1 = (1, 5, 7)$, $C_1 = (-3, 1, 3)$.
$A_2 = (2, 5, 9)$, $B_2 = (5, 8, 12)$, $C_2 = (1, 4, 7)$ are triangular fuzzy numbers.

We observed the new data values $x = 3$, $y = 7$. Use the Mamdani inference rule to find the fuzzy output. Determine the centroid to defuzzify the fuzzy output. [8]

5. Consider the fuzzy rule:

If the apple is red, then it is sweet is fairly true.

The red and sweet are defined by fuzzy sets, respectively as follows:

$$R = 0.7/x_1 + 0.4/x_2 + 0.6/x_3 + 0.5/x_4 + 0.8/x_5$$
$$S = 0.8/y_1 + 0.5/y_2 + 0.6/y_3 + 0.7/y_4$$

Applying the Luckasiewicz fuzzy implication and appropriate quantifiers $g(a) = \sqrt{a}$ for 'fairly.' and $h(a) = a^2$ for 'very,' determine the S_{new} when the new apple input R_{new} observed is very red. [8]