MTL732 (Mathematical Finance) IITD MAJOR EXAM Duration of Examination: 2 hour

April. 2019

Instructions

1. The total number of points is 50 (points are indicated in the margin).

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1. Find the arbitrage opportunity if condition $d < r < u$ is violated in binomial tree model. [5]

2. Define the following

   (a) Futures. [2]

   (b) Predictable investment strategy. [2]

   (c) Find the risk-neutral conditional expectation of $S(3)$ given that $S(2) = 110$ and $r = 0.2$. [2]

   (d) What is an admissible strategy (define each term you use). [2]

   (e) Define Coupon bonds and calculate the price of bond (face value=100, maturity=5 years and coupans of 10 paid annually) after 4 years. [2]

3. A 1 year long forward contract on non-dividend-paying stock is entered into when the stock price is 40 and the risk-free rate of interest is 10% per annum with continuous compounding.

   (a) What are the forward price and the initial value of the forward contract ? [3]

   (b) Six months later, the price of the stock is 45 and the risk-free interest rate is still 10%. What are the forward price and the value of the forward contract ? [3]

4. Find the upper bounds on the prices of

   (a) European call option on stock paying no dividend. [4]

   (b) European put option on stock paying dividend. [4]

5. Let $S(0) = 120$, $u = 0.2$, $d = -0.1$ and $r = 0.1$. Consider a call option with strike price $X = 120$ and $T = 2$. Find the option price and replicating strategies. [5]

6. Let $S(0) = 50$, $r = 5\%$, $u = 0.3$ and $d = -0.1$. Find the price of European call and put with $X = 60$ to be exercised after $N = 3$ time steps. [5]

7. If $S(0) = A(0) = 100$ and $S(1)$ can take take two values,

$$S(1) = \begin{cases} 120 & \text{with probability} \quad 0.25 \\ 80 & \text{with probability} \quad 0.75. \end{cases}$$

If $A(1) = 110$, Compute the price of a put option with $X = 100$. [5]

8. Consider the difference of discrete time stock prices where $t = \frac{n}{N}$, $n = 0, 1, \cdots$ and $w_N(t)$ be corresponding symmetric random walk

$$S_N(t + \frac{1}{N}) - S_N(t) \approx (m + \frac{1}{2}\sigma^2)S_N(t)\frac{1}{N} + \sigma S_N(t)(w_N(t + \frac{1}{N}) - w_N(t)).$$

(a) Show that the continuous time stock prices $S(t)(S(t) = \lim_{N\to\infty} S_N(t))$ satisfies the SDE given below

$$dS(t) = (m + \frac{1}{2}\sigma^2)S(t)dt + \sigma S(t)dW(t).$$

[4]

(b) Also discuss the nature of the process $W(t)$ and its properties. [2]