Department of Mathematics

MTL 763 (Introduction to Game Theory)

Major Exam

Time: 2 hours
Max. Marks: 50

Date: 19/11/17

Note: The exam is closed-book, and all the questions are compulsory.

Q.1 Consider the bimatrix game given below

$$\begin{pmatrix} (2,3) & (3,\alpha) & (2,1) \\ (\alpha,2) & (1,1) & (3,2) \\ (2,2) & (3,1) & (2,\alpha) \end{pmatrix},$$

where $\alpha \in \mathbb{R}$. Compute the set of pure strategy Nash equilibria and Pareto optimal points for all values of α.

(4 marks)

Q.2 Consider the bimatrix game given below

$$A = \begin{pmatrix} 2 & 3 & 2 \\ 4 & 2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \end{pmatrix}.$$

Show that the game has infinite number of Nash equilibria by finding them explicitly.

(5 marks)

Q.3 Consider the following instance of the prisoners' dilemma problem.

P1 \ P2	NC	C
NC	(-4, -4)	$(-2, -x)$
C	$(-x, -2)$	$(-x, -x)$

Find the values of x for which:

(*a*) the profile (C,C) is a strongly dominant strategy equilibrium.
(*b*) the profile (C,C) is a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium.
(*c*) the profile (C,C) is a Nash equilibrium but not a dominant strategy equilibrium.
(*d*) the profile (C,C) is not even a Nash equilibrium.

In each case, say whether it is possible to find such an x. Justify your answer in each case.

(4 marks)

Q.4 Consider the Rock-Paper-Scissor game. Compute the Saddle point equilibrium of the game by using linear programming method.

(6 marks)

Q.5 (*a*) Construct a network formation game with four players which has a unique pairwise stable which is not efficient and strongly stable, and it also has two Pareto efficient networks. Justify your claims with proper arguments.

(3 marks)

(*b*) Consider the 3-players co-author game discussed in the class. Show that all the networks are Nash stable networks. Take a star network with first node as center of the node and find a Nash equilibrium strategy profile in link-announcement game which generates this network.

(3 marks)

Figure 1: Bi-directed triangle network.

Q.6 Consider the network shown in Figure 1. We assume that there are two players both with *source* s and sink t. Player 1 and player 2 want to route 1 and 2 units of traffic *respectively on a particular* path. What can you say about the equilibrium flow in this network game?

(5 marks)

Q.7 Consider a network with source s and destination t. A large population of selfish players *represented* by $[0, 1]$ is traveling from s to t. The costs for the different edges of the network are *given below:*
$(i)\ s \xrightarrow{ax} u$, $(ii)\ u \xrightarrow{1} t$
$(iii)\ s \xrightarrow{1} w$, $(iv)\ w \xrightarrow{ax} t$, $(v)\ u \xrightarrow{0} w$.
What is the range of a for which the equilibrium flow uses all the paths? What is the price of anarchy for $a = 2$?

(5 marks)

Q.8 Consider an n-player network formation game. Each player incurs the cost due to the formation of direct links as well as from indirect links. The cost incurred by a player from direct link is $\alpha > 0$, and the cost from an indirect link is the shortest distance between the nodes. Then, the cost incurred by player i at network g is given by

$$c_i(g) = \alpha \cdot K_i(g) + \sum_{j=1}^{n} d_{i,j}(g),$$

where $K_i(g)$ denote the degree of node i and $d_{i,j}(g)$ denote the shortest distance between i and j at network g. Find at least one pairwise stable network and efficient network for all values of α? What can you say about the price of anarchy of the game?

(7 marks)

Q.9 Consider a Cournot competition among electricity firms over an electricity network comprises of a set of nodes. Let N denote the set of nodes and N_i denote the subset of nodes where firm i has installed its generation facilities. Let I_k denote the set of firms who owns generation facilities at node k. Let x_k^i be the generation quantity for firm i at node k and $(x_k^i)^2$ be its cost of generation. Denote a generation level vector of firm i by $x^i = (x_k^i)_{k \in N_i}$, and a generation level vector at node k by $\hat{x}_k = (x_k^i)_{i \in I_k}$. The price at each node k depends only on the generation quantities of the firms whose generation facilities are installed at node k, and it is determined by the sum of the generation quantities. The price at node $k \in N$ is given by $P_k(\hat{x}_k) = a_k - b_k \sum_{j \in I_k} x_k^j$, where $a_k \in \mathbb{R}$ and $b_k \geq 0$. We assume that the capacity of generation facility of firm i at node k is C_k^i. Formulate the above Cournot competition as a strategic game and show that there exists a Nash equilibrium for the game. Then, consider an instance of this game where there are 4 electricity firms and 3 nodes such that $N_1 = \{1, 2\}$, $N_2 = \{2, 3\}$, $N_3 = \{1\}$, $N_4 = \{2\}$. Formulate the equivalent linear complementarity problem for the above instance of Nash equilibrium problem.

(8 marks)