

Department of Mathematics  
MTL 763 (Introduction to Game Theory)

Major Exam

Time: 2 hours  
Max. Marks: 50

Date: 19/11/17

Note: The exam is closed-book, and all the questions are compulsory.

Q.1 Consider the bimatrix game given below

$$\begin{pmatrix} (2,3) & (3,\alpha) & (2,1) \\ (\alpha,2) & (1,1) & (3,2) \\ (2,2) & (3,1) & (2,\alpha) \end{pmatrix}$$

where  $\alpha \in \mathbb{R}$ . Compute the set of pure strategy Nash equilibria and Pareto optimal points for all values of  $\alpha$ .

(4 marks)

Q.2 Consider the bimatrix game given below

$$A = \begin{pmatrix} 2 & 3 & 2 \\ 4 & 2 & 1 \end{pmatrix}, B = \begin{pmatrix} 2 & 3 & 4 \\ 3 & 2 & 1 \end{pmatrix}$$

Show that the game has infinite number of Nash equilibria by finding them explicitly.

(5 marks)

Q.3 Consider the following instance of the prisoners' dilemma problem.

	P2	NC	C
P1			
NC		(-4, -4)	(-2, -x)
C		(-x, -2)	(-x, -x)

Find the values of  $x$  for which:

- the profile (C,C) is a strongly dominant strategy equilibrium.
- the profile (C,C) is a weakly dominant strategy equilibrium but not a strongly dominant strategy equilibrium.
- the profile (C,C) is a Nash equilibrium but not a dominant strategy equilibrium.
- the profile (C,C) is not even a Nash equilibrium.

In each case, say whether it is possible to find such an  $x$ . Justify your answer in each case.

(4 marks)

Q.4 Consider the Rock-Paper-Scissor game. Compute the Saddle point equilibrium of the game by using linear programming method.

(6 marks)

Q.5 (a) Construct a network formation game with four players which has a unique pairwise stable which is not efficient and strongly stable, and it also has two Pareto efficient networks. Justify your claims with proper arguments.

(3 marks)

(b) Consider the 3-players co-author game discussed in the class. Show that all the networks are Nash stable networks. Take a star network with first node as center of the node and find a Nash equilibrium strategy profile in link-announcement game which generates this network.

(3 marks)

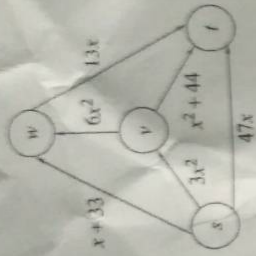


Figure 1: Bi-directed triangle network.

**Q.6** Consider the network shown in Figure 1. We assume that there are two players both with source  $s$  and sink  $t$ . Player 1 and player 2 want to route 1 and 2 units of traffic respectively on a particular path. What can you say about the equilibrium flow in this network game? (5 marks)

**Q.7** Consider a network with source  $s$  and destination  $t$ . A large population of selfish players represented by  $[0, 1]$  is traveling from  $s$  to  $t$ . The costs for the different edges of the network are given below:

- (i)  $s \xrightarrow{ax} u$ , (ii)  $u \rightarrow t$
  - (iii)  $s \xrightarrow{1} w$ , (iv)  $w \xrightarrow{ax} t$ , (v)  $u \rightarrow v$ .
- What is the range of  $a$  for which the equilibrium flow uses all the paths? What is the price of anarchy for  $a = 2$ ? (5 marks)

**Q.8** Consider an  $n$ -player network formation game. Each player incurs the cost, due to the formation of direct links as well as from indirect links. The cost incurred by a player from direct link is  $\alpha > 0$ , and the cost from an indirect link is the shortest distance between the nodes. Then, the cost incurred by player  $i$  at network  $g$  is given by

$$c_i(g) = \alpha \cdot K_i(g) + \sum_{j=1}^n d_{i,j}(g),$$

where  $K_i(g)$  denote the degree of node  $i$  and  $d_{i,j}(g)$  denote the shortest distance between  $i$  and  $j$  at network  $g$ . Find at least one pairwise stable network and efficient network for all values of  $\alpha$ ? What can you say about the price of anarchy of the game? (7 marks)

**Q.9** Consider a Cournot competition among electricity firms over an electricity network comprises of a set of nodes. Let  $N$  denote the set of nodes and  $N_i$  denote the subset of nodes where firm  $i$  has installed its generation facilities. Let  $I_k$  denote the set of firms who owns generation facilities at node  $k$ . Let  $x_k^i$  be the generation quantity for firm  $i$  at node  $k$  and  $(x_k^i)^2$  be its cost of generation. Denote a generation level vector of firm  $i$  by  $x^i = (x_k^i)_{k \in N_i}$ , and a generation level vector at node  $k$  by  $\hat{x}_k = (x_k^i)_{i \in I_k}$ . The price at each node  $k$  depends only on the generation quantities of the firms whose generation facilities are installed at node  $k$ , and it is determined by the sum of the generation quantities. The price at node  $k \in N$  is given by  $P_k(\hat{x}_k) = a_k - b_k \sum_{j \in I_k} x_k^j$ , where  $a_k \in \mathbb{R}$  and  $b_k \geq 0$ . We assume that the capacity of generation facility of firm  $i$  at node  $k$  is  $C_k^i$ . Formulate the above Cournot competition as a strategic game and show that there exists a Nash equilibrium for the game. Then, consider an instance of this game where there are 4 electricity firms and 3 nodes such that  $N_1 = \{1, 2\}$ ,  $N_2 = \{2, 3\}$ ,  $N_3 = \{1\}$ ,  $N_4 = \{2\}$ . Formulate the equivalent linear complementarity problem for the above instance of Nash equilibrium problem. (8 marks)