

Department of Mathematics

Indian Institute of Technology Delhi

MTL 743 FOURIER ANALYSIS: Major

Total marks: 50

Time: 2 hours and 30 Minutes

1. No marks will be provided if appropriate justification is not provided.
 2. Every question is compulsory.
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1. (a) Find the Fourier series of the Function $f(x) = \pi^2 x - x^3$, $x \in [-\pi, \pi]$. [3]

(b) Find the sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^3}$. [2]

2. Let $1 \leq p < \infty$. If $f \in L^1(\mathbb{T})$ and $g \in L^p(\mathbb{T})$ then show that $f * g \in L^p(\mathbb{T})$. [5]

3. Show that the Dirichlet kernel is not a summability kernel. [5]

4. Show that the space of all continuous functions on \mathbb{T} having absolutely summable Fourier series is closed under pointwise product. [5]

5. State and prove Lebesgue's theorem. [5]

6. (a) Find the Fourier transform of $e^{-a|x|}$, $x \in \mathbb{R}$. [3]

(b) For $a, b > 0$, evaluate $\int_0^{\infty} \frac{dx}{(a^2+x^2)(b^2+x^2)}$. [2]

7. Compute the Fourier transform of the locally integrable functions $\sin(2\pi x)$ and $\cos(2\pi x)$. [5]

8. Find the Fourier transform of the derivative of $\chi_{[a,b]}$, where $a, b \in \mathbb{R}$. [5]
9. State and prove Poisson summation formula. [5]
10. Let $\varphi \in L^\infty(\mathbb{R})$. Suppose that X is a subspace of $L^1(\mathbb{R})$ such that $f * \varphi = 0$ for all $f \in X$. Show that $\text{supp}(\widehat{\varphi}) \subseteq Z(X)$. [5]