

Department of Mathematics

Indian Institute of Technology Delhi

MTL 743 FOURIER ANALYSIS: Minor-II

Total marks: 21

Time: 1 hour

1. No marks will be provided if appropriate justification is not provided.
2. Every question is compulsory.

1. Find the Fourier transform of $e^{-\frac{(x-a)^2}{b}}$, $a \in \mathbb{R}$, $b > 0$. [4]

2. If $f \in L^1(\mathbb{R})$, then show that \hat{f} is uniformly continuous. [4]

3. Show that the Schwartz space is dense in $L^p(\mathbb{R})$ for all $1 \leq p < \infty$. [4]

4. Consider the function

$$H(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{if } x < 0 \end{cases} .$$

(a) Compute the derivative of H . [2]

(b) Compute the Fourier transform of H' . [2]

5. Compute the derivative of $\wedge * \varphi$, where $\wedge \in \mathcal{S}'(\mathbb{R})$ and $\varphi \in \mathcal{S}(\mathbb{R})$. [4]