

Please start each question on a new page and write your name and entry number on top of each sheet. Please ensure correct mapping of questions and page number in gradescope.

1. Each of the seven students wrote an informal and a formal essay. The variables recorded were the number of words and the number of verbs:

$y_1$  = number of words in the informal essay

$y_2$  = number of verbs in the informal essay

$x_1$  = number of words in the formal essay

$x_2$  = number of verbs in the formal essay

The data is as follows:

student	$y_1$	$y_2$	$x_1$	$x_2$
1	148	20	137	15
2	159	24	164	25
3	144	19	198	27
4	103	18	128	33
5	119	17	154	21
6	123	13	158	16
7	217	29	214	25

It is assumed that the two samples are from multivariate normal populations. Use the paired test to test the null hypothesis for  $\underline{\mu}_y = \underline{\mu}_x$  at 1% level of significance.

Take a vector  $\underline{q} = S_d^{-1} \underline{\bar{d}}$ , where  $\underline{\bar{d}}$  is the mean of the difference vector  $\underline{d} = \underline{y} - \underline{x}$ , and  $S_d$  is the variance matrix of the difference vector. Test the hypothesis  $H_0 : \underline{q}^T \underline{\mu}_y = \underline{q}^T \underline{\mu}_x$  at 1% level of significance. [7]

2. Four psychological tests were given to 32 boy kids and 32 girl kids. The variables for which the data is collected are

$x_1$  = pictorial inconsistencies

$x_2$  = spatial visualization

$x_3$  = tool recognition

$x_4$  = vocabulary

The mean vectors and covariance matrices of the two random samples are as follows:

$$\bar{x}_b = \begin{pmatrix} 15.97 \\ 15.91 \\ 27.19 \\ 22.75 \end{pmatrix} \quad \bar{x}_g = \begin{pmatrix} 12.34 \\ 13.91 \\ 16.66 \\ 21.94 \end{pmatrix} \quad S_b = \begin{pmatrix} 5.192 & 4.545 & 6.522 & 5.250 \\ 4.545 & 13.18 & 6.760 & 6.266 \\ 6.522 & 6.760 & 28.67 & 14.47 \\ 5.250 & 6.266 & 14.47 & 16.65 \end{pmatrix} \quad S_g = \begin{pmatrix} 9.136 & 7.549 & 4.864 & 4.151 \\ 7.549 & 18.60 & 10.22 & 5.446 \\ 4.864 & 10.22 & 30.04 & 13.49 \\ 4.151 & 5.446 & 13.49 & 28.00 \end{pmatrix}$$

You can assume that the random samples are collected from normally distributed populations  $N(\underline{\mu}_b, \Sigma)$  and  $N(\underline{\mu}_g, \Sigma)$ , where  $b$  stands for boy kids and  $g$  stands for girl kids.

Test the null-hypothesis  $\mu_{b,j} - \mu_{b,j-1} = \mu_{g,j} - \mu_{g,j-1}$ ,  $j = 2, 3, 4$ , at 1% level of significance. [7]

3. We are studying the effectiveness of a basic Maths course in a degree program. We took a random sample of 7 students from the course to assess their attitude toward the course at three time points: prior to the start of the course, during the conduct of the course, and one month after the course is over. Those data are presented below, with higher numbers indicating more positive attitudes toward the course.

student	before	during	after
1	12	6	9
2	8	3	7
3	16	15	14
4	4	3	3
5	9	4	6
6	13	8	10
7	14	11	15

Perform an ANOVA test to determine whether the attitudes for the course differ at the three times of assessment (use  $\alpha = 0.01$ ). You can assume the data at each of the three instances to be normally distributed. [6]