Department of Mathematics

Indian Institute of Technology Delhi

MTL 101 Linear Algebra and Differential Equations: Major

Total marks: 40 Time: 2 hours

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1. Every question is compulsory
2. No marks will be provided for answers without proper justification
3. Questions are printed on both the sides

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1. Determine (with justification) whether the following statements are true or false. [$2 \times 3 = 6$]

   (a) If X_1, X_2 and X_3 are solutions of the system $AX = B$, then $X_1 - 3X_2 + 2X_3$ is not a solution of $AX = 0$, where $A \in M_{m\times n}(\mathbb{R})$ and $B \in M_{m\times 1}(\mathbb{R})$.

   (b) $\mathcal{L}(fg) = \mathcal{L}(f)\mathcal{L}(g)$, where $\mathcal{L}$ denotes the Laplace transform.

   (c) If f and g are differentiable functions such that $f(x) \neq 0$ for every $x \in \mathbb{R}$ and $W(f,g)(x) = 0$ for all $x \in \mathbb{R}$, then f and g are linearly dependent.

2. Let W_1, W_2 be subspaces of $\mathbb{R}^4$ defined by $W_1 = span\,\{(1,2,3,0),(0,1,2,3)\}$ and $W_2 = span\,\{(1,0,-1,1),(2,1,0,-1)\}$.

   (a) For $i = 1, 2$, express the space W_i as the solution space of a system of linear equations in 4 unknowns. [2]

   (b) Find a basis for $W_1 \cap W_2$ and hence $dim(W_1 + W_2)$. [2+1]

3. Using Cayley-Hamilton theorem, find the inverse of the matrix $\begin{pmatrix} 3 & 2 & 1 \\ 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$. [3]

4. Consider the IVP $y' = \frac{y}{\sqrt{1+x}}$; $y(0) = 1$.

   (a) Find the Picard's approximations y_1 and y_2. [3]

   (b) Does the sequence of Picard's approximations converge to the solution? Justify. [1]

5. Using power series method find the general solution of the ODE [4]

$$xy'' - (1+x)y' + y = 0.$$

9 + 2 + 2   6 + 4 + 3   3 + 6 + 1

3 + 2 + 6   2 + 4 + …

6. Solve the following system of differential equations: [5]

$$\begin{aligned} x_1' &= -x_1 + x_2 + 2x_3 \\ x_2' &= -2x_1 + x_2 + 2x_3 \\ x_3' &= -3x_1 + x_2 + 4x_3 \end{aligned}$$

7. *Find the inverse Laplace transform of* [4]

$$\frac{s}{(s-1)(s^2+1)} + \frac{e^{2s}}{(s+3)^3}.$$

8. Solve the IVP [4]

$$2y'' + 10y = 3u(t-12) - 5\delta(t-4); \; y(0) = -1, \; y'(0) = -2.$$

9. Solve the IVP [5]

$$\begin{aligned} x_1' &= 2x_1 + 4x_2 + 2\delta(t-1) \\ x_2' &= x_1 + 2x_2 \\ x_1(0) &= 0, x_2(0) = -2. \end{aligned}$$

$\frac{4}{\sqrt{3}}(2-\sqrt{3}) - \frac{2}{\sqrt{3}}$

$\left(\frac{8-2}{\sqrt{3}}\right) - 4$

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