MAJOR EXAM

PHL100 Electromagnetic Waves and Quantum Mechanics

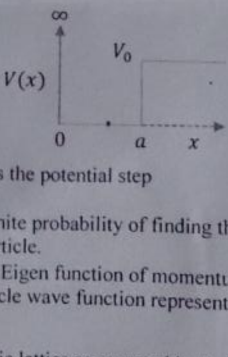
Answer all questions **Time: Two Hours** **Date: 5th May 2014**

All notations have their usual meaning unless otherwise specified. **Marks: 50**

1. A large piece of magnetic material has uniform magnetization **M** and fields $\mathbf{B_0}$ and $\mathbf{H_0}$ inside the material. A small spherical cavity is hollowed out of the material and filled with a diamagnetic material, with magnitude 0.1M magnetization. What are the changes in the fields B and H at the centre of the cavity? (4)
2. From Maxwell's equations, obtain the boundary conditions satisfied by electric and magnetic fields across an interface between two dielectrics that are free of charges and currents. (3)
3. Consider light of intensity 1.35×10^3 W/m^2. What are the magnitudes of electric and magnetic fields and Poynting vector? (3)
4. Calculate the time averaged energy density of an electromagnetic plane wave in a conducting medium. Show that the magnetic contribution always dominates. (5)
5. A mirror needs only a very thin layer coating of silver onto the back of glass. Justify by giving reasons. (2)
6. A particle has the wave function: $\psi(x) = \begin{cases} A\frac{x}{a}, & \text{if } 0 \le x \le a \\ A\frac{(b-x)}{(b-a)}, & \text{if } a \le x \le b \\ 0, & \text{Otherwise} \end{cases}$. Determine A by normalizing the wave function. Where is the particle most likely to be found? (3)
7. (a) If operators A and B are self-adjoint, then show that i[A, B] is also self-adjoint. (3)

   (b) Evaluate $\Delta x \Delta p$ for a particle with wave function: $\psi = \left(\frac{1}{\pi a}\right)^{1/4} e^{-x^2/2a}$ (4)
8. Consider the one dimensional problem of a particle of mass m in a potential $V = \infty$ for $x < 0$; $V = 0$ for $0 \le x \le a$, and $V = V_0$ for $x > a$ (see Fig). Obtain the wave functions and show that the bound state energies (**for** $E < V_0$) are given by: (5)

$$\tan\frac{\sqrt{2mE}}{\hbar}a = -\sqrt{\frac{E}{V_0 - E}}$$



9. A stream of particles of mass m and energy E move towards the potential step $V(x) = 0$ for $x < 0$ and $V(x) = V_0$ for $x > 0$.
   If the energy of the particles $E < V_0$, show that there is a finite probability of finding the particle for $x > 0$. Also, determine the transmitted and reflected flux of the particle. (6)
10. (a) Show that the wave function $\psi(x,t) = e^{-i(kx-\omega t)}$ is an Eigen function of momentum operator. (3)

    (b) Mention the two problems associated with the free-particle wave function represented by,

    $\psi_k(x,t) = Ae^{i(kx - \frac{\hbar k^2}{2m}t)}$ (2)

    (c) Draw the simplified potential of one-dimensional periodic lattice as assumed in the Kronig-Penney model. What is the main outcome of the Kronig-Penney model? (2)

    (d) What is the paradox (in view of classical mechanics) involved in alpha decay? (2)

    (e) Show that the general solution of a particle in the infinite square well returns to its original form after a quantum revival time, $\tau = 4ma^2/\pi\hbar$. (3)

Given: For a magnetized sphere, $\mathbf{B}_{in} = \frac{2}{3}\mu_0\mathbf{M}$;

$c=3\times10^8$ m/s; $\varepsilon_0=8.85\times10^{-12}$ C^2/N.m^2; $\mu_0=4\pi\times10^{-7}$ N/A^2; $\int_0^\infty x^2 e^{-ax^2}dx = \frac{\sqrt{\pi}}{4}a^{-3/2}$