PHL 120: Physics of Materials

Duration: 60 Minutes **Minor I (August 30, 2014)** **Max. Marks: 20**

Q. 1. **(a)** The wave function of a particle at a given time is

$$\psi(x) = \sqrt{\frac{2}{L}}\sin\frac{\pi x}{L} \quad for \quad 0 < x < L$$
$$= 0 \qquad for\ x \le 0 \quad and \quad x \ge L$$

Obtain the probability of finding the particle in the range L/4 < x < 3L/4.

(b) Compute the expectation value of p for the wave function given in part (a). Give physical arguments to explain your answer.

[3+2]

Q.2. An electron moves in the x-direction with a velocity of 3.6×10^{6} m/s. Assuming its velocity is measured to a precision of 1%. **(m_e=9.1×10^{-31} Kg, $\hbar$=1.05×10^{-34} J.s)**

(i) Estimate the minimum possible uncertainty in its position along x-direction?

(ii) Give your comments (with brief explanation) about its motion in the y-direction?

[3+2]

Q.3. **(a)** Check if the function

$$\psi(x) = Ae^{-x/a} \quad for \quad x > 0$$
$$= -Ae^{+x/a} \quad for \quad x < 0$$

is a wave function for a real particle.

(b) Consider a wave function for a particle as

$$\psi(x) = Ae^{ikx} + Be^{-ikx} \quad for \quad x < 0$$
$$= Ce^{-kx} \qquad for \quad x > 0$$

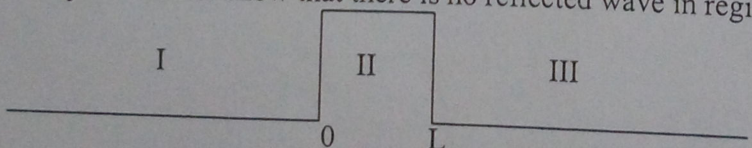
Show that A+B=C

[2+2]

Q.4. **(a)** A particle of mass m is incident on a rectangular barrier of height V_0 extended from x=0 to x=L (see figure below). The energy of the particle is $E = V_0 + \frac{\hbar^2\pi^2}{2mL^2}$. The wave functions in the three regions are:

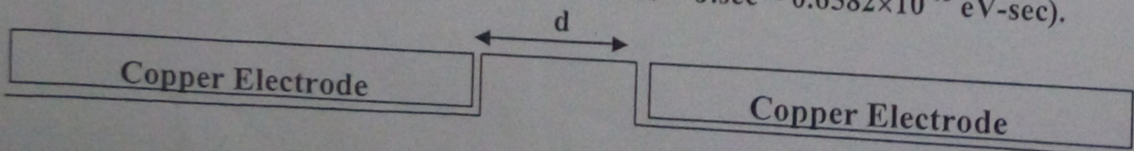
$$\psi_I(x) = A_1e^{ik_1x} + B_1e^{-ik_1x} \qquad for \quad x < 0$$
$$\psi_{II}(x) = A_2e^{ik_2x} + B_2e^{-ik_2x} \quad for \quad 0 < x < L$$
$$\psi_{III}(x) = A_3e^{ik_1x} \qquad for \quad x > L$$

By using the boundary conditions show that there is no reflected wave in region I.



(b) The figure below shows two copper electrodes separated by a small distance (d = 0.5 nm). Make a rough estimate of the probability that an electron incident from the left could tunnel across the gap (which can be represented as a rectangular potential barrier) between the electrodes. The work function of copper (i.e. the minimum energy required to remove an electron from the surface) is 4.7 eV. **(Given m_e=9.31×10^{-31} Kg = 0.5110 MeV/c^2, $\hbar$=1.055×10^{-34} J.sec = 0.6582×10^{-15} eV-sec).**

[4+2]



$A_2\left[1-\frac{k_2}{k_1}\right] + B_2\left[1+\frac{k_2}{k_1}\right]$