Department of Physics, IIT Delhi

PHL755: Statistical and Quantum Optics (Instructor: Kedar Khare)

Date: 22 Nov. 2015

Major Examination

1. The complex analytic signal $z(t)$ corresponding to real valued stationary random process $x(t)$ is given by:

$$z(t) = \frac{1}{2}[x(t) + i\ y(t)].$$

(a) Find an expression for $y(t)$ in terms of $x(t)$.

(b) If $z_1(t)$ and $z_2(t)$ are complex analytic signals, show that:

$$\int_{-\infty}^{\infty} dt\ z_1(t)\, z_2(t) = 0.$$

(5 x 2 = 10 points)

2. The equi-time correlation function for a light field is given by:

$$\Gamma(\vec{r}_1, \vec{r}_2, 0) = f(\vec{r}_1)\ g(\vec{r}_2)\ .$$

Show that the light field is fully spatially coherent everywhere. **(10 points)**

3. The total energy H of the free space electromagnetic field is given by:

$$H = 2\sum_{k}\sum_{s} \omega^2\, |u_{ks}(t)|^2$$

where $u_{ks}(t) = c_{ks}\exp(-i\,\omega\, t)$.

(a) Using the definitions $q_{ks}(t) = [u_{ks}(t) + c.c.]$ and $p_{ks}(t) = -i\omega\,[u_{ks}(t) - c.c.]$, show that $q_{ks}(t)$ and $p_{ks}(t)$ satisfy Hamilton's equations of motion.

(b) Replacing $u_{ks}(t)$ by the operator $\sqrt{\frac{\hbar}{2\omega}}\ \hat{a}_{ks}(t)$, find an expression for the operator $\hat{H}$.

(5 x 2 = 10 points)

4. The coherent state is defined as:

$$|\alpha> = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n}\frac{\alpha^n}{\sqrt{n!}}\,|n>$$

(a) Show that two coherent states $|\alpha>$ and $|\beta>$ are not orthogonal when $\alpha \neq \beta$.

(b) Show that the coherent state may be expressed as:

$$|\alpha> = \exp(\alpha\,\hat{a}^+ - \alpha^*\,\hat{a}\,)\,|0>$$

(5 x 2 = 10 points)

5. (a)The density operator describing the state of electromagnetic field can be represented as:

$$\hat{\rho} = \int d^2\alpha\ \ \phi(\alpha)\ |\alpha><\alpha|$$

Evaluate the expectation value of the operator $(\hat{a}^+\hat{a})^3$ when the state of the light field corresponds to a single mode laser.

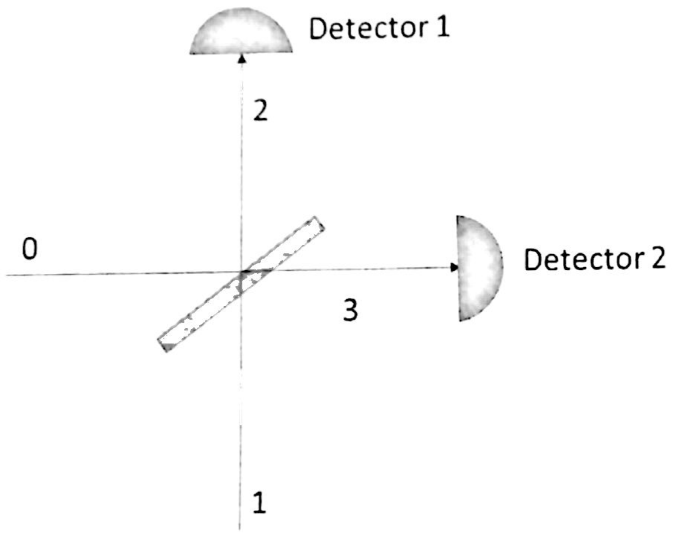
(b) A modified form of coherent state as described by:

$$|\,\alpha, \phi > = \exp\left[\frac{\phi}{2}\,(\hat{a}^2 - \hat{a}^{+2})\right]\,|\,\alpha >$$

Find the expectation values of the operators $\hat{Q} = \frac{\hat{a}^+ + \hat{a}}{2}$ and $\hat{P} = i\,\frac{\hat{a}^+ - \hat{a}}{2}$ in this state.

(5 x 2 = 10 points)

6. Consider a beamsplitter arrangement with two detectors:



(a) Evaluate the coincidence counting rate at the two detectors if a state $|\,1_0\,, 1_1 >$ is incident at the input of the beamsplitter.
(b) Find an expression for the differential photodetector signal if the input state consists of a squeezed vacuum at the port 0 and a coherent state $|\,\alpha >$ at the port 1.

(5 x 2 = 10 points)

Useful identity:

$$\exp(x\,\hat{A})\;\hat{B}\;\exp(-\,x\,\hat{A}) = \hat{B} + x\,[\hat{A},\hat{B}] + \frac{x^2}{2!}\,[\hat{A},[\hat{A},\hat{B}]] + \cdots$$

If $[\hat{A},[\hat{A},\hat{B}]] = 0$, $\quad \exp(\hat{A} + \hat{B}) = \exp(\hat{A})\,\exp(\hat{B})\,\exp(-[\hat{A},\hat{B}]/2)$