Indian Institute of Technology, Delhi
Minor 1: PHL 755 Statistical and Quantum Optics

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1. (a) Define the characteristic function $C(\omega)$ associated with a probability density function $p(x)$. **(5 points)**
(b) Find the characteristics function associated with the Poisson random distribution

$$p(n) = \frac{e^{-<n>} <n>^n}{n!}$$

(5 points)
(c) If n_1 and n_2 are two independent Poisson random variables with means $< n_1 >$ and $< n_2 >$ respectively, show that the random variable $N = n_1 + n_2$ is also a Poisson random variable. Find the expected mean of N. **(5 points)**

2. (a) Consider a wide sense stationary random process $x(t)$. How is the spectral density $S_x(\nu)$ of the process related to its autocorrelation $\Gamma_x(\tau)$? **(5 points)**
(b) The random process $x(t)$ above serves as an input to a linear system with impulse response $h(t)$. Find the autocorrelation of the output $y(t)$ of the system given by:

$$y(t) = \int_{-\infty}^{\infty} dt'\, x(t') h(t-t')$$

if the autocorrelation $\Gamma_x(\tau) = A\delta(\tau)$ for some constant A. **(5 points)**
(c) What is the spectral density of the process $y(t)$? **(5 points)**

3. (a) The propagation law for mutual intensity is given by:

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$$J(\vec{r}_1, \vec{r}_2) = (\frac{\bar{k}}{2\pi})^2 \int\int d^2\vec{r'}_1 d^2\vec{r'}_2\, J(\vec{r'}_1, \vec{r'}_2)\, \frac{\exp[i\bar{k}(R_1 - R_2)]}{R_1 R_2}$$

Here $J(\vec{r}_1, \vec{r}_2) =< E^*(\vec{r}_1)E(\vec{r}_2) >$ is the equi-time field correlation. Obtain the far-zone form of the van Cittert-Zernike theorem for an incoherent quasi-monochromatic source. **(5 points)**
(b) A rectangular incoherent source of radius a is placed in plane $z = 0$ with its center at the origin. A Young's double slit experiment is performed by putting a screen with two tiny pinholes in the plane $z = z_0$ which is in the far-zone from the source. Find the separation of pinholes for which fringes with good visibility can be observed. **(5 points)**

4. (a) A Young's double slit experiment uses a point illuminating source with spectral density:

$$S(\nu) = \delta(\nu - \nu_0 + \frac{\Delta\nu}{2}) + \delta(\nu - \nu_0 - \frac{\Delta\nu}{2}).$$

The two pinholes being illuminated by this source are located in the $z = 0$ plane at $(a, 0)$ and $(-a, 0)$ respectively. How does the visibility of fringes near the point $(a, 0, z)$ change as a function of distance z ? (You may assume $z >> a$.) **(5 points)**
(b) How will your answer of part (a) change if one of the two spectral lines is filtered out with a bandpass filter? **(5 points)**

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