Department of Physics, IIT Delhi

PYL - 100: Major portion (Quantum Mechanics)

Total = 33 marks

ALL questions are compulsory and should be answered in the Major Test Answer Booklet. Extra sheets, if used for this portion, should be attached with the answer booklet used.

1. If $\hat{D}_x$ is defined as $\frac{\partial}{\partial x}$ and $\psi(x) = A\sin\frac{n\pi x}{a}$, then
   (a) Find the results of the operation of (i) $\hat{D}_x$ on $\psi(x)$, (ii) $\hat{D}_x^{\,2}$ on $\psi(x)$.
   (b) Which one of the above is an eigen value problem? What is the eigen value?

   [1+2 = 3 Marks]

2. (a) Write down the time-*independent* Schrödinger equation in one dimension. How is the Hamiltonian operator $\hat{H}$ expressed here?
   (b) Using this $\hat{H}$, evaluate $[\hat{x}, \hat{H}]$.
   (c) Given a wave function: $\phi(x) = \frac{e^{-\alpha x^2}}{\sqrt{N}}$ and an operator $\hat{A} = \left(\frac{d^2}{dx^2} - bx^2\right)$, where N, α and b are constants. If $\phi(x)$ is an eigen function of $\hat{A}$, what should be the value of 'b'? **[2+1+2 = 5 Marks]**

3. A particle of mass 'm' which moves freely inside an **infinite potential well** of length 'a' has the following initial wave function:
$$\Psi(x,0) = \frac{A}{\sqrt{a}}\sin\left(\frac{\pi x}{a}\right) + \sqrt{\frac{3}{5a}}\sin\left(\frac{3\pi x}{a}\right) + \frac{1}{\sqrt{5a}}\sin\left(\frac{5\pi x}{a}\right).$$
   (a) Identify the *stationary states* involved in the above wave function.
   (b) Find 'A' so that $\Psi(x,0)$ is normalized.
   (c) If a measurement of energy is carried out what are the values that will be found? What will be their corresponding probability?
   (d) Calculate the average energy. **[1+2+3+1 = 7 marks]**

4. A particle of mass 'm' is in a 1-D finite potential well of the form given below:
$$V(x) = \begin{cases} 0 & for\,|x| < \frac{a}{2} \\ V_0 & for\,|x| \geq \frac{a}{2} \end{cases}$$
   Given the depth of the potential well, V_0 lies in the range: $\frac{\pi^2\hbar^2}{2ma^2} < V_0 < \frac{2\pi^2\hbar^2}{ma^2}$.
   (a) Using graphical method, determine how many bound states are possible.
   (b) Draw the form of the appropriate eigenfunctions corresponding to these two bound states.

   [3+2 = 5 Marks]

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5. A particle of mass 'm' is confined in a **finite potential well** extending from $x = -a/2$ to $x = a/2$. It is in the **ground state**. At a particular instant, the value of the ground state eigen function at $x = 0$ is $\Psi = M$, where 'M' is a constant. And at $x = a/4$ it is $\Psi = \frac{\sqrt{3}}{2}M$.

(a) Calculate the ground state energy of the particle.

(b) Calculate the depth of the potential well.

[3+3 = 6 Marks]

6. Consider a **barrier potential**, defined as

$$V = 0 \quad \text{for} \quad x < 0,$$
$$= V_0 \quad \text{for} \quad 0 < x < a,$$
$$= 0 \quad \text{for} \quad x > a.$$

(a) For a particle incident from the left with energy E (where $E < V_0$), *derive* the 'general' solutions of the wave function in the three regions. (Do NOT evaluate the constants).

(b) Draw the potential barrier. Schematically plot the form of the wave function solutions obtained above in the three regions.

(c) The transmission coefficient in region III is given as:

$$T = \left[1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2(\gamma a)\right]^{-1}, \quad \text{where } \gamma = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}.$$

State the condition under which we can approximately write: $T \approx e^{-2\gamma a}$.

[3+3+1 = 7 Marks]