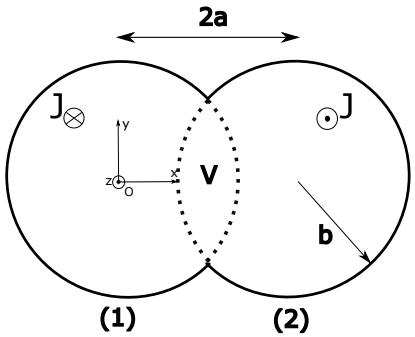
Major (PYL100)

Time: 2 Hours (9:30 - 11:30AM) *5 Sep. 2020* **Max. Marks 50**

Show all the intermediate steps of your calculations. The use of unfair means, or abetment will lead to the forfeiture of your entire test score and disciplinary action.
We must receive your answer script submission by 12:00PM.

1. A thin hemispherical shell of radius R has a uniformly distributed surface charge density σ_s. Find $\boldsymbol{E}$ at the center of the sphere circumscribing the hemisphere, and indicate it with a diagram. **[3 marks]**

2. Copper wires (1) and (2), carrying opposing uniform currents of magnitude J, have a cross section given by two intersecting circles of radius b with centers separated by $2a$. However, note that the *lens-shaped* region V bounded by the dotted lines consists of vacuum. Given the coordinate system below, find $\boldsymbol{B}$ inside the region V. **[3 marks]**

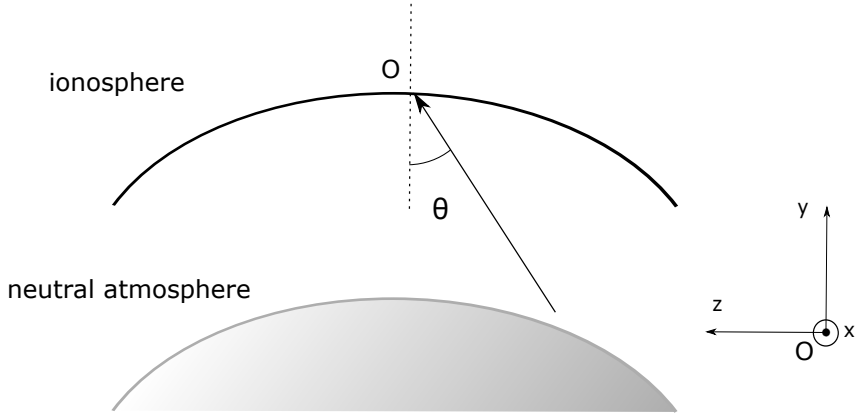


3. Recall Minor-II. Instead we have an s-polarized plane EM wave of amplitude E_0 and wave vector k, incident at an angle θ upon the ionosphere (at point O) which is modelled as a perfect conductor. Assume that the neutral atmosphere has properties of the vacuum. Given the coordinate system below, obtain **[4 marks]**

   (a) (1.5 marks) the time-averaged Poynting vectors of all the individual rays.

   (b) (0.5 marks) the reflection coefficient R.

   (c) (2 marks) the total time-averaged Poynting vector.



4. A particle's Hamiltonian ($\hat{H}$) has eigen values 1, 2 and 3 corresponding to orthonormal eigen states, $|\phi_1\rangle$, $|\phi_2\rangle$ and $|\phi_3\rangle$, respectively. Initially (at time t = 0), the particle is found to be in a state given by $|\psi\rangle = \frac{1}{\sqrt{2}}|\phi_1\rangle + \frac{1}{\sqrt{6}}|\phi_2\rangle + \frac{1}{\sqrt{6}}|\phi_3\rangle$. Show all steps in the following. [8 marks]

   (a) What is the expectation value of $\hat{H}$ in the initial state? [3 marks]

   (b) What is the state of the particle at time 10 seconds? [3 marks]

   (c) What is the expectation value of $\hat{H}$ in the new state? [2 marks]

5. Find out the eigen functions and eigen values of the momentum of a particle of mass m moving inside an infinite one dimensional potential well of width *a*. [4 marks]

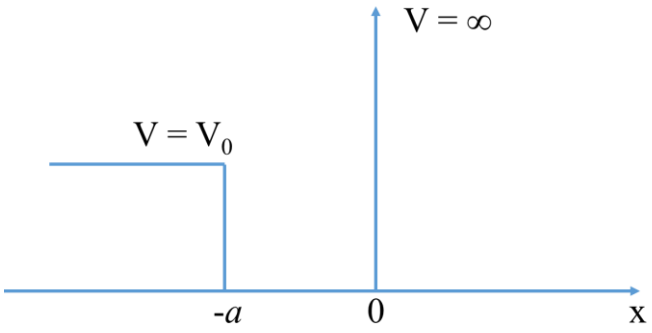
6. The free particle in one-dimension can be described by a wave function of a wave packet as $\psi(x) = ae^{-\frac{x^2}{b^2}} e^{i\frac{p}{\hbar}x}$. Here, *a* and *b* are constants, while *p* is the momentum of the particle. Calculate the uncertainties in the position and the momentum. [6 marks]

   Given some useful formulae:

   $\int_{-\infty}^{+\infty} e^{-ax^2+bx}dx = \sqrt{\frac{\pi}{a}}\, e^{b^2/4a}$; $\int_{-\infty}^{+\infty} x^{2n}e^{-x^2}dx = \frac{1\times3\times5....(2n-1)\sqrt{\pi}}{2^n}$ for $n = 0,1,2,\ldots$

   $\int_{-\infty}^{+\infty} x^{2n+1}e^{-x^2}dx = 0$ for $n = 0,1,2,\ldots$

7. A quantum particle of mass m and energy $E < V_0$ is moving in one-dimensional space having the potential function as shown. Write complete answers of all the parts below. [14 marks]



   (a) (3 marks) Set up and explain the equation of motion for the particle in the entire space available.

   (b) (6 marks) Find first three normalized eigen functions and eigen energies. Also, show a sketch within the potential function.

   (c) (2 marks) Are there bound states? If yes, how many? If no, why?

   (d) (3 marks) If *a* is reduced to *a*/2 and V_0 is increased to $2V_0$, find the first three normalized eigen functions and eigen energies. Draw a new sketch.

8. A metal piece having dimensions 1mm x 1mm x 1mm, has 2.54 x 10^{19} free electrons. Solve all parts below. [8 marks]

   (a) (2 marks) Calculate the Fermi energy of the metal.

   (b) (1 mark) Calculate the phase space volume occupied by the electrons?

   (c) (1 mark) Calculate the number of states occupied by the electrons?

   (d) (3+1 marks) If 5*eV* photons are used to excite the metal surface, the electrons are ejected out with KE = 1*eV*. Find the total phase space volume and the total number of states possible.