ReMajor (PYL100)

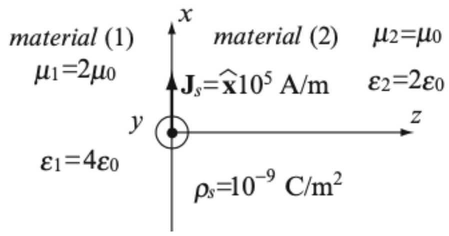
Time: 2 Hours (9:00 - 11:00AM) *22 Sep. 2020* **Max. Marks 50**

Show all the intermediate steps of your calculations. The use of unfair means, or abetment will lead to the forfeiture of your entire test score and disciplinary action.
We must receive your answer script submission by 11:30AM.

1. A tiny circular loop of radius $R \ll L$, centered at the origin with its axis along $\hat{\boldsymbol{z}}$, carries a current I. Obtain the magnetic flux linked with an identical loop with the same radius, having center at $(0, 0, L)$ and axis along $\hat{\boldsymbol{z}}$. **[5 marks]**

2. An interface between two materials (1) and (2) with properties shown below contains both a current density given as $\boldsymbol{J}_s = \hat{\boldsymbol{x}} 1 \times 10^5$ A/m and a uniform surface charge density $\rho_s = 1 \times 10^{-9}$ C/m^2. The static magnetic field intensity and static electric field intensity (which are independent of each other) in material (1) are **[5 marks]**

$$\boldsymbol{H}_1 = 1 \times 10^5 \hat{\boldsymbol{x}} + 1 \times 10^5 \hat{\boldsymbol{y}} - 1 \times 10^5 \hat{\boldsymbol{z}} \ [A/m], \quad \boldsymbol{E}_1 = 100\hat{\boldsymbol{x}} + 20\hat{\boldsymbol{y}} - 100\hat{\boldsymbol{z}} \ [V/m]$$



   (a) (2.5 marks) Find $\boldsymbol{E}$ in (2)

   (b) (2.5 marks) Find $\boldsymbol{B}$ in (2).

3. The wave function of a particle confined in one-dimension within the region ($0 \le x \le 4$), is given by $\psi(x,t) = A\, sin\left(\frac{\pi x}{4}\right) e^{-i\omega t}$. Find the potential $V(x)$ in which the particle is moving. What is the probability of finding the particle in the region ($2 \le x \le 3$)? [3 + 3 marks]

4. A quantum system in 1d is described by three particular solutions of the Schrodinger equation, i.e., $\psi_1(x)$, $\psi_2(x)$ and $\psi_3(x)$ and the corresponding eigen energies $E_1 = \hbar$, $E_2 = 2\hbar$ and $E_3 = 3\hbar$, respectively.

   (a) Construct a general solution for the system and normalize it. Show all steps. [4 marks]

   (b) Calculate the probability density at 'x' and a given time, (i) $t = 0$ sec, (ii) $t = 5$ sec. Show all the steps. [5 marks]

   (c) Plot the time-dependent probability density with complete details at a particular 'x' and from $t = 0$ sec to $t = 2\pi$ sec. [5 marks]

5. A particle of mass m is moving inside an infinite potential well ($V = 0$ for $-L/2 < x < L/2$). Construct a wave packet to describe the particle simultaneously present in first three states and show the normalization condition. [5 marks]

6. For a particle of mass m in a 1d potential well, it can be found with certain probabilities in two states having energies as $E_1 = \hbar/2$ and $E_2 = 3\hbar/2$ corresponding to eigen functions $\psi_1(x) = Ae^{-\frac{m\omega}{2\hbar}x^2}$ and $\psi_2(x) = Bxe^{-\frac{m\omega}{2\hbar}x^2}$. Here, A and B are constants. What is the value of $\Delta x \Delta p$ in the two states? Show all the steps. [5 marks]

7. Consider a potential function, $V(x) = \alpha\delta(x)$, where α is a constant and δ is Delta function. How many bound states are possible for the particle of mass m. Also, obtain the eigen energies and the corresponding eigen functions for the bound states. [5 + 5 marks]