PYL100: Electromagnetic Waves & Quantum Mechanics

I Semester 2014-15

Minor - I | **Time: 1 hour**

29-8-2014 | *Answer all questions* | **Max. Marks: 25**

NOTE: All symbols used have the usual meaning. Wherever applicable, result should be given in vector form and in respective units. You may use the given Formula Sheet.

1. Consider a sphere of radius a, made of inhomogeneous dielectric; the dielectric constant (or relative permittivity) variation in the sphere is given by $\varepsilon_r(r) = \dfrac{1}{(1+A\,r)}$, where A is a constant and r is the radial distance from the origin. A charge q is placed at the centre of the sphere.
   (a) Find the *electric displacement*, *electric field* and *polarization* vectors as a function of r. **(3)**
   (b) Also, find the *volume density* and *surface density* of the bound charges. **(2)**

2. Consider a long solid-cylinder of radius a that carries a magnetization $\vec{M} = \beta\, s^2 \hat{\varphi}$,where β is a constant, s is the radial distance from the axis of the cylinder, and $\hat{\varphi}$ is the usual unit vector in the cylindrical coordinate system (s, φ, z).
   (a) Find the total *surface current* and *volume current*. **(3)**
   (b) Find the magnetic field $\vec{B}$ due to the magnetization $\vec{M}$, both inside and outside the cylinder. **(2)**

3. The electric field associated with a plane electromagnetic wave in a particular medium is given by $\vec{E}(\vec{r},t) = \hat{x}\, E_0 \exp\left[i\left(a(\sqrt{3}y+z) - \pi\times 10^{15} t\right)\right] V\, m^{-1}$, where E_0 is the real amplitude, and a is a constant. Determine -
   (a) The direction of propagation of the wave. **(1)**
   (b) The free-space wavelength of the wave. **(1)**
   (c) The magnetic field $\vec{B}$ associated with the wave. **(2)**
   (d) The Poynting vector corresponding to the wave. **(1)**

4. (a) The x- and y-components of the electric field of a polarized beam of light are given by

$$E_x = E_0 \cos(kz - \omega t), \qquad E_y = \sqrt{2} E_0 \cos\left(kz - \omega t + \frac{\pi}{2}\right).$$

   If the above light beam is passed through a quarter-wave plate (QWP), whose refractive index for x-polarized wave is less than that for the y-polarized wave, obtain the state of polarization of the emergent light (i.e. after passing through the QWP). **(3)**

   (b) Consider a pair of *crossed polarisers* with a QWP in between them. If I_0 is the intensity of an unpolarised light beam, incident on the above combination, what would be the output intensity when - **(i)** the *fast axis* of the QWP is parallel to the *pass axis* of the input polarizer, **(ii)** the *fast axis* of the QWP makes an angle of 45^0 with the *pass axes* of the polarisers. **(2)**

5. (a) The *phase velocity* of an electromagnetic wave in a particular medium is given by $v_p = \sqrt{\dfrac{a}{\omega}}$, where a is a constant and ω is the angular frequency. Determine the *group velocity* of the wave, and the ratio of phase velocity to group velocity. **(3)**

   (b) Consider the planar interface between two linear dielectric media with permittivity ε_1 and ε_2. There is a uniform electric filed E_1 in Medium 1, and the electric field lines make an angle θ_1 with the plane of the interface. Applying relevant boundary conditions, determine the direction of the electric field lines in Medium 2. **(2)**

Physical Constants: $c = 3 \times 10^8$ m/s, $\mu_0 = 4\pi\times 10^{-7}$ N/A^2, $\varepsilon_0 = 8.854 \times 10^{-12}$ C^2/ N m^2, $e = 1.6 \times 10^{-19}$ C