1 Multiple-choice questions [$1 \times 10 = 10$ marks]

9

The alphabetical letter (A,B,C or D) corresponding to your answer should be written clearly within the square brackets beside the question. Any ambiguity or untidiness will be awarded zero points. All symbols have their usual meanings unless otherwise indicated. There is no negative marking.

1. The *electrostatic* regime requires [D]

   A. $\int \nabla \cdot \boldsymbol{E}\, d\tau = 0$
   B. $\int \boldsymbol{E} \cdot dl = 0$
   C. $\nabla \cdot \boldsymbol{E} = 0$
   D. $\nabla \times \boldsymbol{E} = 0$

2. The divergence of $\hat{\boldsymbol{r}}/r^2$ where $\boldsymbol{r}$ is the radial vector in spherical coordinates is [C]

   A. ∞
   B. $2\pi\delta^3(\boldsymbol{r})$
   C. $4\pi\delta^3(\boldsymbol{r})$
   D. 0

3. The potential difference between two points, both lying *inside* a uniformly charged spherical shell holding a charge Q, and radius R is [A]

   A. 0
   B. $\frac{Q}{4\pi\epsilon_0 R}$
   C. positive
   D. negative

4. The electric displacement $\boldsymbol{D}$ outside a wire carrying a uniform line charge λ is [B]

   A. 0
   B. $\frac{\lambda}{2\pi r}\hat{\boldsymbol{r}}$
   C. ∞
   D. $\frac{\lambda}{2\pi r^2}\hat{\boldsymbol{r}}$

5. Two point charges -1×10^{-6} C and 4×10^{-6} C separated by 3 m have a zero field location from one of the charges of [C]

   A. 1 m

   B. 1 m

   C. 3 m

   D. 4 m

6. The magnetic field lines due to a straight wire carrying a current are [B]

   A. straight

   B. circular

   C. parabolic

   D. elliptical

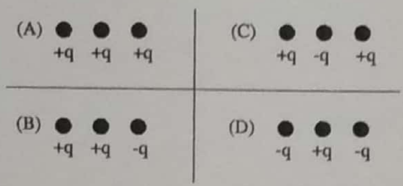
7. Rank the work required to assemble the charge distributions where the separations between the charges is identical in all cases. [B]

   A. $A < B < C = D$

   B. $C = D < B < A$

   C. $A < B = C = D$

   D. $B = C = D < A$



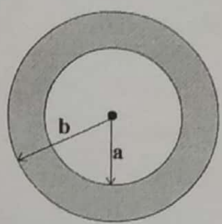
8. A net charge of $+Q$ is transferred to a spherical conducting shell of inner radius a and outer radius b. A point charge $-10q$ is placed in the center of the shell. What is the charge density on the outside of the conducting shell? [A]

   A. $\frac{-10q}{4\pi b^2}$

   B. $\frac{Q}{4\pi b^2}$

   C. $\frac{Q+10q}{4\pi b^2}$

   D. $\frac{Q-10q}{4\pi b^2}$



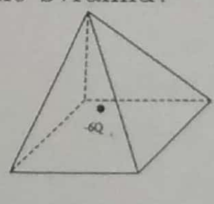
9. A charge $q = -6Q$ rests inside a pyramid which has a lateral base perimeter of a units and a lateral base height of b units. What is the total electric flux through the pyramid? [B]

   A. $\frac{-3Q}{4\pi\epsilon_0 b^2}$

   B. $\frac{-6Q}{\epsilon_0}$

   C. $\frac{-6Qa^2}{\epsilon_0 b^2}$

   D. $\frac{-3Qa^2}{\epsilon_0 b^2}$



10. Two *coaxial* circular, and tightly wound coils with identical turns n per unit length carry the same current I but in opposite directions. The magnitude of the magnetic field B at a point on the axis midway between the coils is [A]

    A. 0

    B. The same as that produces by one coil

    C. Twice that produced by one coil

    D. Half that produced by one coil

2 Descriptive questions [15 marks]

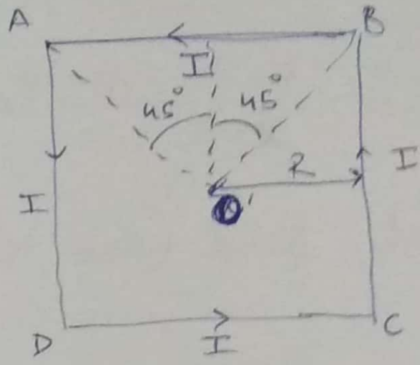
Write your solution in the space provided on the question paper itself. There is part marking. However, if your solution is illegible, or doesn't straightforwardly lead to your final answer you will receive a zero.

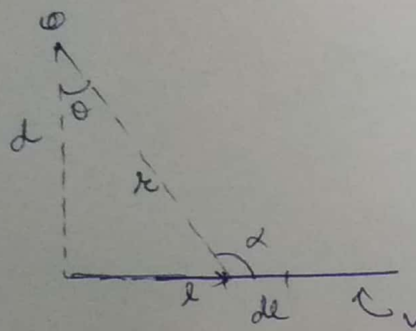
1. **Magnetic fields due to current loops** [5 marks]

5

(a) (3 marks) Find the magnetic field at the center of a square loop, which carries a steady current I. Let R be the distance from center to side.

deriving the magnetic field (B) due to a wire at a distance d.





wire [current = I]

3

$$B(O) = \frac{\mu_0 I}{4\pi}\int \frac{\overline{dl}\times\overline{r}}{r^3} \quad \text{(Biot-Savart law)}$$

$\overline{dl}\times\overline{r} = dl\, r \sin\alpha = dl\, r\cos\theta$

$l = d\tan\theta$

$dl = (d)\sec^2\theta\, d\theta$

$r = \dfrac{d}{\cos\theta}$

$$B(O) = \frac{\mu_0 I}{4\pi d}\int_{\theta_1}^{\theta_2} \cos\theta\, d\theta$$

$$= \frac{\mu_0 I}{4\pi d}\left[\sin\theta_2 - \sin\theta_1\right]$$

for side AB (see figure)

$$|B(O')| = \frac{\mu_0 I}{4\pi R}\left[\frac{1}{\sqrt{2}} - \left(-\frac{1}{\sqrt{2}}\right)\right]$$

$\theta_1 = 45°,\ \theta_2 = -45°$

$$B(O') = \frac{\sqrt{2}\mu_0 I}{4\pi R}\hat{z} \quad \text{(assuming a direction of current)}$$

~~for AD~~ similarly for sides AD, DC, BC

$$B(O') = \frac{\sqrt{2}\mu_0 I}{4\pi R}\hat{z}$$

Net field at O' = $\dfrac{4\times\sqrt{2}\mu_0 I}{4\pi R}\hat{z} = \dfrac{\sqrt{2}\mu_0 I}{\pi R}\hat{z}$ Tesla

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(b) (1 mark) Find the field at the center of a regular n-sided polygon, carrying a steady current I. Again, let R be the distance from the center to any side.

for an n sided polygon,

$\theta_2 = \pi/n$, $\theta_1 = -\pi/n$

R π/n π/n

$$B_{(0)} = \frac{\mu_0 I}{4\pi R}\left(\sin \pi/n - \sin(-\pi/n)\right) \longrightarrow \left[\text{this is due to 1 side}\right]$$

$$= \frac{\mu_0 I}{2\pi R}\sin(\pi/n)$$

Net field (due to n sides) $= \dfrac{\mu_0 I n}{2\pi R}\sin(\pi/n)$ Tesla

1

(c) (1 mark) Check that your formula reduces to the field at the center of a circular loop, in the limit $n \to \infty$.

Net field (for n-sided polygon) $= \dfrac{\mu_0 I n}{2\pi R}\sin(\pi/n)$ T

$$\lim_{n\to\infty} B = \lim_{n\to\infty} \frac{\mu_0 I n}{2\pi R}\sin(\pi/n)$$

evaluating the limit at $n \to \infty$

1

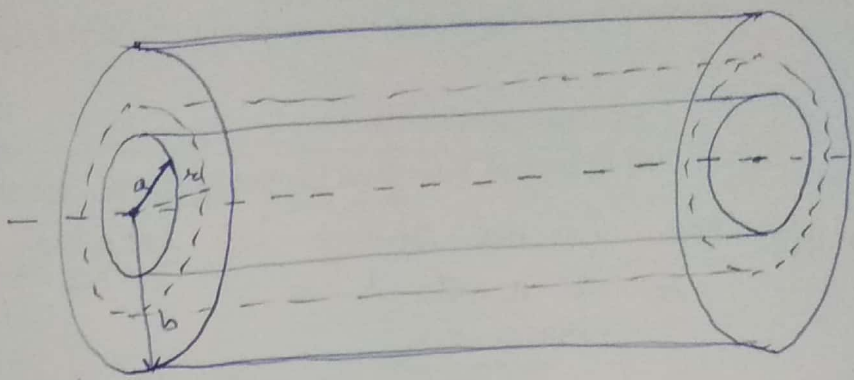
$$\lim_{n\to\infty} B = \lim_{n\to\infty} \frac{\mu_0 I n}{2\pi R}\,\frac{(\pi/n)\,(\sin(\pi/n))}{(\pi/n)}$$

$$= \frac{\mu_0 I}{2R} = \text{field at center of a circular loop.}$$

2. Find the capacitance per unit length of two coaxial metal cylindrical tubes, of radii a and b, where $(b > a)$. [5 marks]

$4\frac{1}{2}$

let the magnitude of charges on the surfaces of cylinder be = Q



- First finding the potential (V) between the cylinders.

use Gauss's law

$$\int E \cdot da = \frac{q_{enclosed}}{\epsilon_0}$$

imagine a gaussian surface in the form of cylinder of radius $r \rightarrow a < r < b$

$$E \cdot 2\pi r l = \frac{Q}{\epsilon_0}$$ where l = length of cylinder

$$\vec{E} = \frac{Q}{\epsilon_0 2\pi r l}\hat{r}$$

$V_b - V_a = -\int_b^a \vec{E} \cdot dl$

$$V = -\int E \cdot dl$$

$$= \frac{-Q}{2\pi\epsilon_0 l}\int_a^b \frac{1}{r}dr$$

$$= \frac{Q}{2\pi\epsilon_0 l}\ln\left(\frac{b}{a}\right)$$

using $Q = CV$

$$\frac{C}{l} = \frac{Q \times 2\pi\epsilon_0 l}{Q\ln(a/b)\, l}$$

$$= 2\pi\epsilon_0 \ln(b/a)$$

$$= \frac{2\pi\epsilon_0}{\ln(b/a)}$$

3. A thick spherical shell (inner radius a, outer radius b) is made of dielectric material with a "frozen-in" polarization [5 marks]

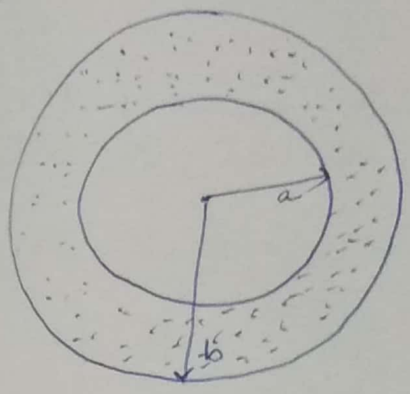
$$P(r) = \frac{k}{r}\hat{r}$$

where k is a constant and r is the distance from the center. (There is no free charge in the problem.)

(a) (2 marks) Locate all the bound charge, and use Gauss's law to calculate the field it produces.

- all the bound charge is located between $r = a$ and $r = b$

  $\Rightarrow a < r < b$



- field due to bound charge E

  for $r < a \Rightarrow E = 0$ 1

  for $a < r < b$

  ~~[illegible]~~ Gauss's law $\Rightarrow \nabla \cdot E = \frac{\rho}{\epsilon_0}$ (ρ = charge density)

  $$\rho = \int \rho \cdot d\tau = \int \frac{k}{r} 4\pi r^2 dr = \frac{k \times 4\pi r^2}{2} = \boxed{k \times 2\pi r^2}$$

?

(b) (3 marks) Find $\boldsymbol{D}$, and then get $\boldsymbol{E}$.

We know

$$\nabla \cdot D = \rho_f$$

$$\rho_f = 0$$

$$D = \epsilon_0 E + P$$

$$E = \frac{D - P}{\epsilon_0}$$

?

$$P(r) = \frac{k}{r}(\hat{r})$$