MidTerm (PYL101)
December 27, 2020

Time: 2 Hours (9:30AM - 11:30AM) **Max. Marks 50**

Show all the intermediate steps of your calculations. The use of unfair means, or abetment will lead to the forfeiture of your entire test score.
We must receive your answer script submissions by 12:00PM.
Please read "Points" as "Marks".

1. Verify Stokes' theorem for $\boldsymbol{A} = (2x-y)\hat{\boldsymbol{x}} - 2yz^2\hat{\boldsymbol{y}} - 2zy^2\hat{\boldsymbol{z}}$ on the upper hemispherical surface of a sphere of radius 2 centered at the origin. **[5 marks]**

2. A uniform line charge of density λ lies along the entire periphery (two straight sides and a curved one) of a quarter sector of a circle of radius a, centered at the origin which lies in the first quadrant of the (x, y)-plane. **[5 marks]**
   (a) (3 points) Find $V(0, 0, z)$.
   (b) (2 points) Can one determine $\boldsymbol{E}$ from the $V(0, 0, z)$ above? Explain your answer.

3. A long cylindrical ideal conductor of $r = a$ (connected to zero potential) is covered with a material of dielectric constant ϵ out to $r = b$. A uniform volume charge density ρ_0 is distributed throughout the dielectric. **[5 marks]**
   (a) (2 points) Calculate the potential everywhere inside the charge layer.
   (b) (2 points) Calculate the potential outside the charge layer.
   (c) (1 point) Plot the potential in space.

4. Using the Biot-savart law, find the magnetic field at the center of a square loop, where a is the distance from its center to the side and the loop carries a steady current I, explain all the intermediate steps with proper figure. **[4 marks]**

5. The vector potential of a single dipole moment is given below:

$$A(r) = \frac{\mu_0}{4\pi}\frac{\mathbf{m}\times\hat{r}}{r^2}$$

   Find the potential of a magnetized object at certain distance in terms of bound currents and explain their physical significance. **[4 marks]**

6. Find the relation between permeability of magnetic material and free space. Also, write one basic difference between paramagnetic and ferromagnetic materials. Draw the hysteresis loop for a ferromagnetic material. **[4 marks]**

7. **[6 marks]**
   (a) (3 points) Write the physical interpretation of all the Maxwell's equations and the Poynting's theorem.
   (b) (3 points) Prove that the work done by magnetic force is zero.

8. Consider the following oscillating electric field asssociated with an electromagnetic wave in vacuum. **[4 marks]**

$$\mathbf{E}(z,t) = E_0 cos(3\omega t) cos(3kz)\hat{x}$$

   (a) (2 points) Find the corresponding magnetic field.

   (b) (2 points) Find the averaged Poynting flux and comment on the result.

9. A linearly polarized light with its polarization in x-direction is propagating in z-direction and is incident normally onto a material having a refractive index n_R for right-handed circularly polarized light and n_L for left handed circularly polarized light. Calculate the polarization of the reflected light. **[4 marks]**

10. A plane polarized EM wave with polarization along $\hat{x}$ and traveling along z-direction in a dielectric medium of refractive index n is reflected at normal incidence from the surface (in x-y plane) of a conductor whose refractive index is $n_2 = n(1 + i\rho)$. Find the phase change suffered by its electric field vector as a result of reflection. **[4 marks]**

11. An electromagnetic wave in a cold collisional plasma follows the following dispersion relation

$$k = \frac{\omega}{c}\left(1 - \frac{\omega_p^2}{\omega^2\left(1 + \frac{i\nu}{\omega}\right)}\right)^{1/2}$$

    If the plasma is charaterized by the plasma frequency $\omega_p = 1.01 \times 10^{12}$ Hz and the collision frequency $\nu = 10^7$ Hz, **[5 marks]**

   (a) (3 points) Deduce the expressions for the skin depth for the electromagnetic waves in low ($\omega \ll \nu$) as well as high ($\omega \gg \nu$) frequency regime.

   (b) (2 points) Calculate the skin depth for $\omega = 10^3$ Hz and $\omega = 10^{12}$ Hz using the above expressions.