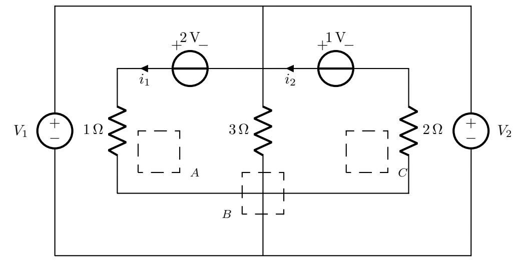
Minor-II (PYL100)

Time: 1 Hour (9:30AM - 10:30AM) *16 Aug. 2020* **Max. Marks 25**

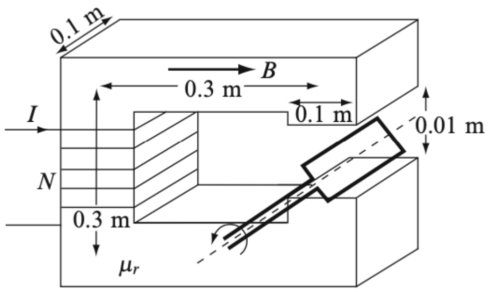
Show all the intermediate steps of your calculations. The use of unfair means, or abetment will lead to the forfeiture of your entire test score.
We must receive your answer script submissions by 11:00AM.

1. A time-varying $\boldsymbol{B}$ of magnitude $100t$ T points out of the page and uniformly threads *only* the interior of the square regions A, B and C, each having an area $0.1\,\mathrm{m}^2$. Assume that all the resistors are tiny, wires perfectly conducting, voltmeters ideal, self-flux due to the currents in the wires negligible, and that the entire apparatus is stationary. For $t > 0$, **[4 marks]**
   (a) (2 marks) find the currents i_1 and i_2.
   (b) (1 mark) find the values recorded by the voltmeters V_1 and V_2.
   (c) (1 mark) what would the voltmeters measure if $\boldsymbol{B}$ was static?



2. Demonstrate whether the force experienced by a point charge subject to an electromagnetic field is *conservative* or not. If not, clearly state the circumstances under which it *is* conservative. **[2 marks]**

3. The following setup has $N = 1000$ (uniformly wound all around the iron core (linear, $\mu_r = 1000$) but not shown for clarity) turns and is powered by a sinusoidal current (where $I(t = 0) = 0$) of amplitude 10 A and frequency 60 Hz. The loop measures $10\ mm \times 10\ mm$ and rotates at 5000 rpm, and at $t = 0$ its surface plane normal is vertical. Assume $\boldsymbol{B}$ in the gap is uniform (no fringing) and perpendicular to the iron surfaces. **[3 marks]**

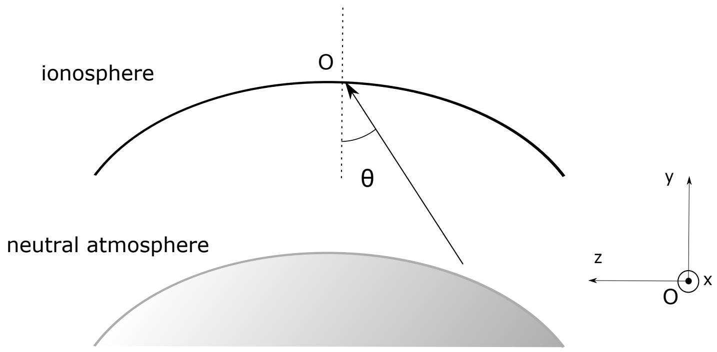


   (a) (1 mark) Find $\boldsymbol{B}$ in the gap.
   (b) (2 marks) Calculate the emf of the loop.

4. Electromagnetic (EM) waves through vacuum **[2 marks]**
   (a) (1 mark) When are such waves said to be plane-polarized?
   (b) (1 mark) Find the condition when the following EM wave is plane-polarized.

$$\boldsymbol{E} = E_{m1}\cos(\omega t - kz)\hat{\boldsymbol{x}} + E_{m2}\cos(\omega t - kz + \theta)\hat{\boldsymbol{y}}$$

5. A p-polarized plane EM wave having an electric field amplitude 200 V/m and frequency 5 GHz propagates in the neutral atmosphere. It's incident at $\theta = 30^\circ$ upon the ionosphere (at point O) which is modelled as a perfect conductor. Assume that the neutral atmosphere has properties of the vacuum. Given the coordinate system below, find in the neutral atmosphere, **[4 marks]**



   (a) (2 marks) the *total* $\boldsymbol{E}$ and $\boldsymbol{B}$.

   (b) (2 marks) the *time-averaged* Poynting vector.

6. A proton is bound inside the nucleus of radius $r_0 = 5 \times 10^{-15}$ m. Estimate the binding potential energy for the proton. Explain why an electron can not reside inside the nucleus. **[1+1=2 marks]**

7. For a one-dimensional harmonic oscillator of mass m, calculate the expectation values of the kinetic energy and the total energy when it is in the second excited state described by $E_2 = \frac{5}{2}\hbar\omega$ and $\Psi_2(x,t) = \left(\frac{\alpha}{\pi}\right)^{1/4} \frac{1}{\sqrt{2}}(2\alpha x^2 - 1)e^{-ax^2/2}e^{-iE_2 t/\hbar}$, where $\alpha = \frac{m\omega}{\hbar}$. The symbols have their usual meaning. How much is the uncertainty in the position of the particle when it is in the ground state? **[2+1=3 marks]**

8. A particle is constrained to move in one-dimension with a probability of 1/3 for being in the region $(-a-b, -a+b)$ and 2/3 for being in the region $(a-b, a+b)$. Here, a and b are real numbers. Construct a possible normalized wave function for the particle. Draw a sketch of the wave function also. **[2+1=3 marks]**

9. Consider a wave function, $\psi(x) = Ax^2 e^{-x^3/\sqrt{2}}$, where A is a constant. This describes well a particle's motion inside a confining potential in the region, $-10 < x < 10$. Calculate the probability of finding the particle inside half of the region, *i.e.*, $-10 < x < 0$? **[2 marks]**