The possession of a mobile phone, ... entire test score. Ensure ... or abetment will lead to the forfeiture of your entire test score. ... pages, with all 6 sides printed.

All answers must be written in the space provided on the question paper itself.

Name: POORVA GARG Entry No.: 2017EE30540 Group No.: 3B.

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1 Multiple-choice and objective questions [11 marks]

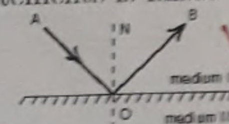
The alphabetical letter (A,B,C or D) corresponding to your answer should be written clearly within the square brackets beside the question, and the answers corresponding to the objective questions in the space provided right below them. Any ambiguity, or untidiness will be awarded zero points. All symbols have their usual meanings unless otherwise indicated. There is **no** *negative marking.*

1. A p-polarized electromagnetic wave of intensity I_0 is obliquely incident on a vacuum-dielectric interface ($z = 0$) at an angle of incidence equal to Brewster's angle. The refractive index of the dielectric is 2. The intensity of the transmitted wave is [A]
   A. I_0
   B. $2I_0$
   C. $I_0/2$
   D. $3I_0/4$
2. An amplitude modulated wave of carrier frequency ω propagates along $\hat{z}$ in a conductor of $\epsilon_r = 9 - \frac{\omega_p^2}{\omega^2}$. Its group velocity at $\omega = \frac{\omega_p}{\sqrt{5}}$ is [A]
   A. $c/2$
   B. $c/9$
   C. $c/18$
   D. $2c/9$
3. An E.M. wave of amplitude A is normally incident from vacuum on a dielectric. The amplitude of the transmitted wave is $\frac{A}{2}$. The relative permittivity of the dielectric is [A]
   A. 9
   B. 2
   C. 4
   D. 1.5

4. A bar magnet is moved at a steady speed of 1 m/s towards a coil of wire which is connected to a centre-zero galvanometer. The magnet is now withdrawn along the same path at 0.5 m/s. The deflection of the galvanometer is in the [B]

   A. same direction as previously, with the magnitude of the deflection doubled
   B. opposite direction as previously, with the magnitude of the deflection halved
   C. same direction as previously, with the magnitude of the deflection halved
   D. opposite direction as previously, with the magnitude of the deflection doubled

5. The diagram shows total internal reflection. Which of the following statements is false? [A]



   A. $\angle AON$ must be the critical angle
   B. $\angle AON = \angle BON$
   C. the speed of light in medium II is greater than that in medium I
   D. if $\angle AON$ were increased, there would still be total internal reflection

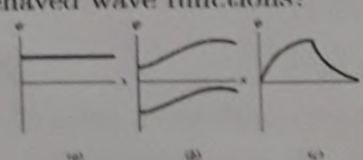
6. The probability of finding a particle described by a wave function in a distance dx around the point x is given by: [C]

   A. ψ^2
   B. $|\psi|^2$
   C. $\psi^*\psi dx$
   D. $\psi\psi^*$

7. Which of the following are eigenfunctions of the operator $\frac{\partial^2}{\partial x^2}$? Note that c and m are both real constants. [C]

   A. c $logx$
   B. cx^2
   C. ce^{-mx}
   D. $\frac{c}{x}$

8. Which of the wave functions shown below are not well-behaved wave functions? [C] ABC



   A.
   B.
   C.

9. Write down the time-dependent Schrödinger equation: Considering in 1 dim.

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + V(x,t)\psi(x,t) = i\hbar\frac{\partial}{\partial t}\psi(x,t)$$

10. The expectation value of position x of a particle trapped in a potential well of length L is

$$\frac{L}{2}$$

11. The quantum mechanical operator corresponding to kinetic energy is

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \quad \text{or} \quad -\frac{\hbar^2}{2m}\nabla^2$$

2 Descriptive questions

Write your solution in the space provided on the question paper itself. There is part marking. However, if your solution is illegible, or doesn't straightforwardly lead to your final answer you will receive a zero.

1. A plane polarized electromagnetic wave travelling in a dielectric medium of refractive index n is reflected at normal incidence from the surface of a conductor. Find the phase change undergone by its electric field vector if the refractive index of the conductor is $n_2 = n(1 + i\rho)$ [4 marks]

3

$$\beta = \frac{n_2}{n} = 1 + i\rho$$

$$r_{\parallel} = \frac{1-\beta}{1+\beta} = \frac{-i\rho}{2+i\rho} = \frac{-i\rho(2-i\rho)}{4+\rho^2} = \frac{-\rho^2 - 2i\rho}{4+\rho^2}$$

$$\cancel{\frac{E_{0R}}{E_{0I}} = \frac{-\rho^2 - 2i\rho}{4+\rho^2}}$$

$$r_{\parallel} = \frac{E_R e^{i\delta_R}}{E_I e^{i\delta_I}} = e^{i(\delta_R - \delta_i)} = \frac{-\rho^2 - 2i\rho}{4+\rho^2}$$

$$\arg(r_{\parallel}) = \text{phase change} = \pi + \tan^{-1}\left(\frac{2\rho}{\rho^2}\right)$$

$$= \pi + \tan^{-1}\left(\frac{2}{\rho}\right) \quad \underline{\text{Ans}}$$

2. Consider a thick conducting slab (conductivity σ) exposed to a ... with peak amplitudes E_0 and B_0. Calculate the Poynting vector within the slab, averaged in time, over one wave period. Consider σ large, i.e., $\sigma >> \omega\epsilon_0$ **[4 marks]**

$$S = \frac{1}{\mu_0}\left(Re(E) \times Re(B) \right)$$

4

$$= \frac{1}{\mu_0} E_0 e^{-kz} \cos i(kz - \omega t + \delta_E)\ B_0 e^{-kz} \cos i(kz - \omega t + \delta_B)\hat{z}$$

$$\int_0^{2\pi} \cos(\theta+\phi)\cos\phi \, d\phi = \frac{1}{2}\cos\theta$$

$$S = \frac{1}{2\mu_0} E_0 B_0 e^{-2kz} \cos(\delta_B - \delta_E)\hat{z}$$

$$= \frac{1}{2\mu_0} E_0 B_0 e^{-2kz} \frac{k}{K}\hat{z}$$

where $K = k + ik$

also $\frac{B_0}{E_0} = \frac{K}{\omega}$

$$= \frac{1}{2\mu_0} E_0^2 \frac{K}{\omega} e^{-2kz} \frac{k}{K}\hat{z}$$

$$= \frac{k}{2\mu_0\omega} E_0^2 e^{-2kz}\hat{z}$$

Now $k = \omega\sqrt{\frac{\mu\epsilon}{2}}\left(\sqrt{1+\left(\frac{\sigma}{\epsilon\omega}\right)^2} - 1\right)^{1/2}$

since $\left(\frac{\sigma}{\epsilon\omega}\right)^2 >>> 1$

$k = \omega\sqrt{\frac{\mu\epsilon}{2}\frac{\sigma}{\epsilon\omega}} = \sqrt{\frac{\omega\mu\sigma}{2}}$

$$= \frac{1}{2\mu_0\omega}\sqrt{\frac{\omega\mu_0\sigma}{2}} E_0^2 e^{-2kz}\hat{z}$$

similarly

$k = \sqrt{\frac{\omega\mu\sigma}{2}}$

$$= \sqrt{\frac{\sigma}{8\mu_0\omega}} E_0^2 e^{-2kz}\hat{z}$$

$$= \sqrt{\frac{\sigma}{8\mu_0\omega}} E_0^2 e^{-\sqrt{2\omega\mu_0\sigma}\, z}\hat{z}$$

Ans

Name, Entry No. and Group No.:

3. A particle is confined to move within a potential of width L. Using the uncertainty principle, calculate the minimum energy of the particle. [2 marks]

~~Δx~~

For minimum energy of the particle, we need to take Δx to be maximum

$$\Delta x = L$$

Heisenberg's uncertainty principle

$$\Delta p \, \Delta x \geq \frac{\hbar}{2}$$

$$\Delta p \geq \frac{\hbar}{2 \Delta x} = \frac{\hbar}{2L}$$

1

$$\Delta E = (\Delta p) c \geq \frac{\hbar c}{2L}$$

non-rel.

Since E can't be greater than its variation, min

$$E \geq \Delta E$$

$$E \geq \frac{\hbar c}{2L}$$

$$\geq \frac{hc}{4\pi L}$$

Ans

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Solving the time-independent Schrödinger equation, write down the wave function for a free particle. Also plot the probability density with distance x for the free particle. [4 marks]

For free particle, $V(x) = 0$ or V_0 (constant). Let's take $V(x) = 0$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi$$

3

$$\frac{\partial^2\psi}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0$$

1

$$\psi = Ae^{ikx} + Be^{-ikx} \quad \text{where } k = \sqrt{\frac{2mE}{\hbar^2}}$$ Ans

1

where A and B are arbitrary constants

For probability density

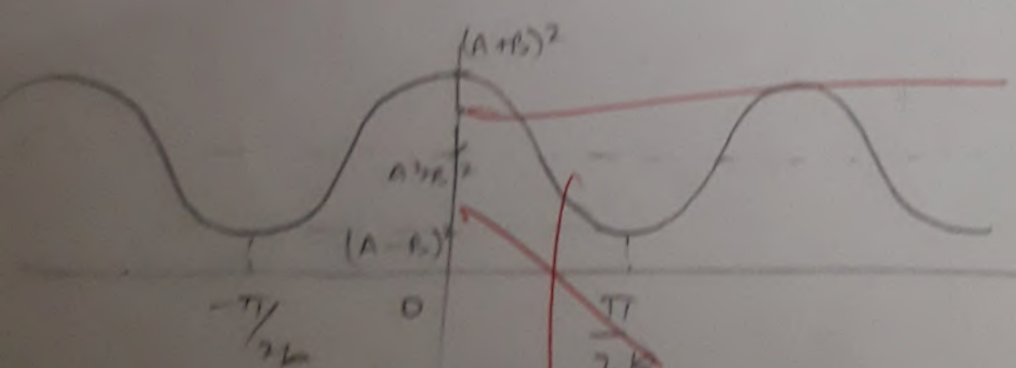
1

$$P(x,t) = \psi^* \psi$$

$$= (Ae^{ikx} + Be^{-ikx})(Ae^{-ikx} + Be^{ikx})$$

$$= A^2 + B^2 + AB(e^{2ikx} - e^{-2ikx})$$

$$= A^2 + B^2 + 2AB(\cos(2kx))$$



Ans