Major (PYL101)

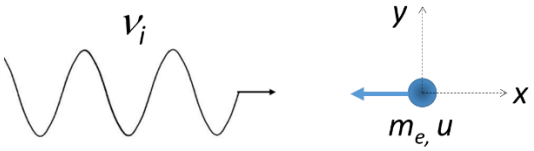
Time: 2 Hours (9:30 - 11:30AM) **Date**: Feb 10, 2021 **Max. Marks**: 50

Show all the intermediate steps of your calculations/analytical solutions. Standard notations have their usual meaning. The use of unfair means or abetment will lead to the forfeiture of your entire test score and disciplinary action. The exam ends at 11.30 AM. You get 20 min as buffer time to compile and submit your answer script as a single pdf file. We must receive your answer script before 11.50 AM. No submission will be considered after 11.50 AM.

1. A solar cell is a device which converts solar light into electricity. It can be thought of as a pn-junction diode, which, upon light illumination, creates current in an external circuit connected to it. Light creates electron-hole pairs in the semiconducting material of the solar cell. List any three processes/events here, which need quantum mechanical principles to understand them. Explain. **[4 marks]**

2. Consider a hydrogen-like bound system with a fixed $+e$ heavy core and a $-e$ electron revolving around it. Here, e is magnitude of electronic charge. In two consecutive measurements, the electron is found to be in two stable orbits at r_1 and $r_2 = 2r_1$ distances, respectively, away from the core. The corresponding energies are E_1 and $E_2 = E_1/4$, respectively. E_1 is the least possible energy of the system. Calculate the ratio between the de Broglie wavelengths of the electron, when it is in these two orbits. **[6 marks]**

3. A photon with initial frequency ν_i hits an electron coming from the opposite direction with speed u, as shown below. After the elastic collision, the electron continues to move in the same direction but with reduced speed $u/2$.



   (a) Find the direction in which the photon gets scattered. **[2 marks]**

   (b) Calculate the energy of the scattered photon. **[5 marks]**

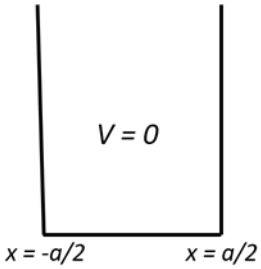
4. The quantum mechanical wave function of a particle is expressed in terms of the superposition of three orthonormal wave functions as $|\phi> = A\,|\psi_1> + \frac{1}{\sqrt{5}}\,|\psi_2> + i\,\frac{1}{\sqrt{7}}\,|\psi_3>$, where, $|\psi_1>$, $|\psi_2>$ and $|\psi_3>$ are the eigen functions of an operator $\hat{O}$ such that $\hat{O}|\psi_n> = (n^2+1)|\psi_n>$, where n = 1, 2, 3.

   (a) Find the value of A. **[2 marks]**

   (b) If you do a measurement of the observable for $\hat{O}$ on $|\phi>$, what value will you get? **[3 marks]**

   (c) What will be the outcome if the measurement is performed on $|\psi_2>$? **[2 marks]**

5. A particle of mass m is confined to move in an infinite potential well in the region, $-a/2 \le x \le a/2$, as shown below.



(a) Write down the boundary conditions satisfied by the wave function of the particle. **[1 mark]**

(b) Find the ground state and first excited state wave functions of the particle. **[3 marks]**

(c) What is the probability to find the particle in the interval -*a*/4 to *a*/4 when it is in the first excited state? **[3 marks]**

(d) If the particle jumps from the first excited state to the ground state, how much energy would be released? (Given, mass of the particle = 4 times the electron mass and $a = 10$ Angstrom) **[3 marks]**

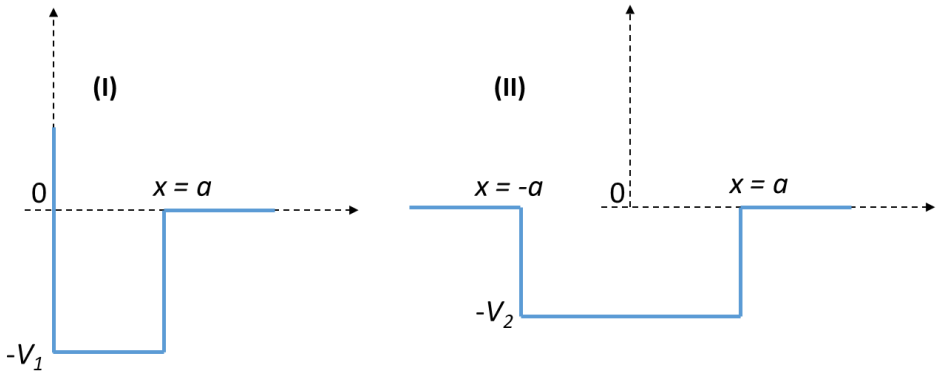
6. At time $t = 0$, a free particle in quantum mechanical state is described by the wave function $\psi(x) = \sqrt{\frac{\alpha}{\pi}}\ e^{-\frac{\alpha x^2}{2}}$, where α is postive real constant.

(a) Find the probability density of the particle with momentum $2\hbar k$ at any time *t*. Here, *k* is the wave vector. **[3 marks]**

(b) Find the mean energy of the particle at any time *t*. **[3 marks]**

(You may require to use $\int_{-\infty}^{\infty} e^{-(ax^2+bx)}\,dx = \sqrt{\frac{\pi}{a}}\ e^{\frac{b^2}{4a}}$)

7. Consider the following one-dimension potential wells.



(a) Derive the conditions for V_1 and V_2 for which these potential wells support bound states. **[6 marks]**

(b) What would be the energy of the particle such that well (I) supports only ONE bound state? What would be that for well (II)? **[2 marks]**

(c) Are the bound states of well (I) also the bound states of well (II)? Explain. **[2 marks]**