
INDIAN INSTITUTE OF TECHNOLOGY DELHI

DEPARTMENT OF PHYSICS

MAJOR EXAM (PYL101)

Date: 3th March 2022, Time: 10:00 – 12:00 Hrs., Max. Marks: 50

- *There are total 8 questions in this question paper.*
- *All questions are compulsory. Use NEW PAGE for each question.*
- *Marks for each question are indicated in brackets.*
- *Some useful constants & conversions are given at the end of the paper*

Question 1.

A 1.2 MeV energy photon collides head-on with a free electron (initially at rest). If the photon is scattered backwards ($\phi = 180^o$), calculate the kinetic energy of the scattered electron. (5)

Question 2.

A beam of electrons is incident normally at the surface of a crystal. The first diffraction maxima occur at 30^o with respect to incident beam. If the inter-atomic separation in the crystal is 2.2 Å, calculate the kinetic energy of the electrons. (4)

Question 3.

(i) Show that two eigenfunctions of a Hermitian operator are orthogonal to each other if the corresponding eigenvalues are unequal. (4)

(ii) Show that:

$$\left[\hat{A}_1,[\hat{A}_2,\hat{A}_3]\right] - \left[\hat{A}_2,[\hat{A}_1,\hat{A}_3]\right] = [\hat{A}_1,\hat{A}_2]\hat{A}_3 + \hat{A}_3[\hat{A}_2,\hat{A}_1]$$ (4)

Question 4.

(i) If $\hat{A}$ is an operator such that $\hat{A}\psi = \psi^2 - 2\psi - 3$, then find out $\hat{A}^2\psi$. (2)

(ii) For a particle with one-dimensional normalized wavefunction:

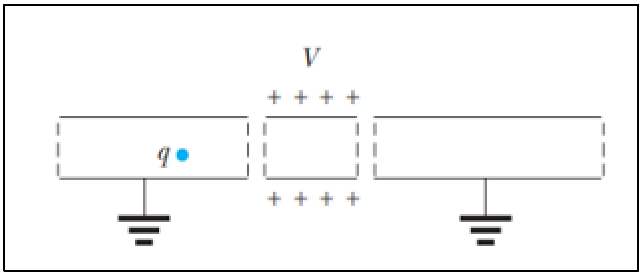
$$\psi(x) = \frac{1}{\sqrt{2}}\, x e^{-\frac{x}{2}} \qquad if\ x > 0,$$
$$\psi(x) = 0 \qquad \dots elsewhere.$$

Calculate the value of σ_x. [Hint: $\sigma_A = \sqrt{\langle(\Delta A)^2\rangle}$] (6)

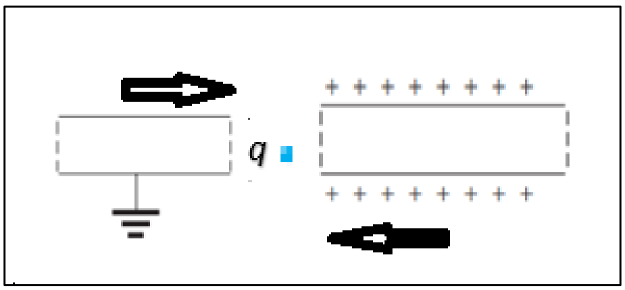
<u>Question 5.</u> (Short answers)

(i) Identify the ideal potentials from, either the physical examples of representative aligned metallic cylinder-arrangements that can experimentally demonstrate the dynamics of the charged particles OR from the corresponding probability density functions. Qualitatively draw the potential and the energy of the microscopic particle, relative to the potential it is experiencing. (3)

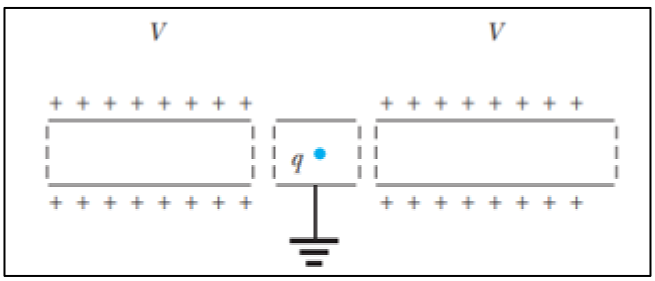
(a)



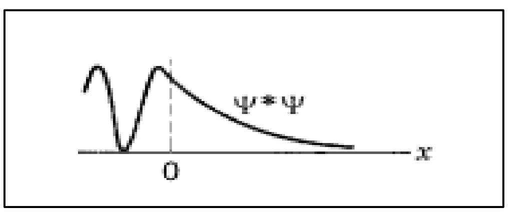
(b)



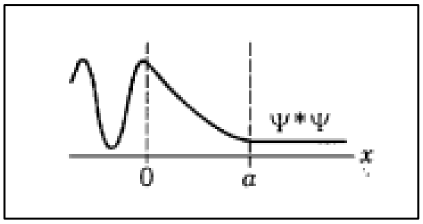
(c)



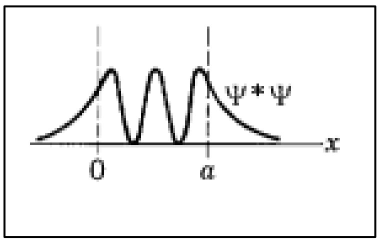
(d)



(e)



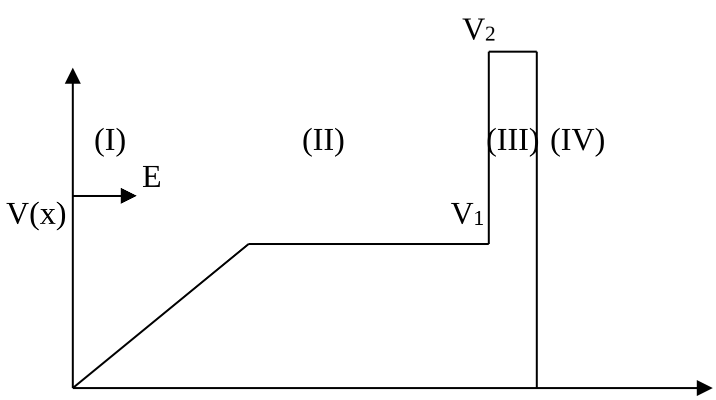
(f)



(ii) If the electron having de Broglie's wavelength at 1.21 x 10^{-8} cm is confined in a 1-D box, how far apart must the walls of the box be when 5 loops of the de Broglie wave span the distance from one wall to the other? (Give your answer in Å) (2)

(iii) Sketch the real part of wavefunction for the given potential, in each zone (I), (II), (III) and (IV) The energy of the particle is $V_2 > E > V_1$. Give pointwise reasons to justify the nature of your sketch with regards to wavelength and amplitude of Ψ. (4)

Answer specifically about λ (qualitative comparison among regions) and $\psi^*\psi$ in each region.



Question 6.

In a step barrier problem with $E < V_0$, the penetration into the forbidden region is associated with the wave nature of the particle. Using uncertainty principle show that the penetration distance Δx is consistent with the uncertainty in defining the location or time coordinate of the particle. We know that the probability density in the x > 0 region is $|\psi|^2$ proportional to $e^{-2k_2 \Delta x}$ and $k_2 = \sqrt{[2m(V_0 - E)]} / \hbar$. (5)

Question 7.

Although an excited atom can radiate at any time from t = 0 to t = ∞ , the average time after excitation at which a group of atoms radiates is called the lifetime, τ, of a particular excited state. **(a)** If $\tau = 1.0 \times 10^{-8}$ s, use the uncertainty principle to compute the line width $\Delta\nu$ of light emitted by the decay of this excited state **(b)** If the wavelength of the spectral line involved in this process is 500 nm, find the fractional broadening $\Delta\nu/\nu$. (4)

Question 8.

An electron that is scattering from a negatively ionized gas atom in the "plasma" of a gas discharge tube can be approximated as being incident upon a rectangular barrier of height V_0= 10 eV and thickness $\boldsymbol{a} = 1.8 \times 10^{-10}$m. Evaluate the transmission coefficient T as a function of the total energy E of the electron, with specific comments about its transmission probability under the conditions asked. We know for $E < V_0$ the Transmission coefficient is given by (7)

$$T = \left[1 + \frac{1}{4}\frac{V_0^2}{E(V_0 - E)} \sinh^2\left(\frac{a}{\hbar}\sqrt{2m(V_0 - E)}\right)\right]^{-1} \quad \sim \quad \mathbf{16\,(E / V_0)\,[1-(E / V_0)]\, e^{-2k_2 a}}$$

And for $E > V_0$, it is

$$= \left[1 + \frac{V_0^2}{4E(E - V_0)} \sin^2\left(a\sqrt{2mV_0/\hbar^2}\sqrt{E/V_0 - 1}\right)\right]^{-1}$$

- **(a)** Show that for $E \ll V_0$, the opacity of the barrier is given by $2mV_0a^2 / \hbar^2$
- **(b)** What will happen if **(i)** $E/V_0 \ll 1$, **(ii)** $E/V_0 = 0.5$, **(iii)** $E > V_0$ **(iv)** $E_n = V_0 + n^2\pi^2\hbar^2 / 2ma^2$
- **(c)** Make a rough qualitative sketch of T (on Y axis) Vs E / V_0 on X-axis (additional ½ mark will be given if you can specify the limits of X-& Y axis correctly from the given data)

1 Å = 10^{-10} m,

1 eV = 1.6 x 10^{-19} J,

1 MeV =10^{6} eV,

1 W = 1 J/s

c = 3 x 10^{8} m/s,

h = 6.6 x 10^{-34} J.s,

Electron's mass = m_e = 9.1 x 10^{-31} kg

Electron's rest mass energy = m_ec^2 = 0.51 MeV,

Bohr radius = a_0 = 5.3 x 10^{-11} m,

Electron charge e = 1.6 x 10^{-19} C,

Boltzmann Constant k = 1.38 x 10^{-23} J/K