Name: TATSAM Entry No.: 2022MT61969

Instructions: There are total six questions All questions are compulsory. Show all intermediate steps. Justify all answers. Clearly specify the question you answer.

Q 1. Let $E(\lambda)d\lambda$ be the radiation emitted per second by a blackbody in the wavelength range λ and $\lambda + d\lambda$.

(a) Plot $E(\lambda)$ versus λ for the temperatures 200 K and 100 K on a same plot and answer the following questions.

(b) Qualitatively mention whether the peak position in $E(\lambda)$ versus λ graph shifts towards higher λ or lower λ with increasing temperatures.

(c) Calculate the ratio of the peak positions, i.e., $\lambda^{max}_{200\,\mathrm{K}}/\lambda^{max}_{100\,\mathrm{K}}$

(d) Calculate the ratio of the area under the $E(\lambda)$ versus λ curves for two temperatures.

[2+1+1+1=5 marks]

Q 2. An electron moves in x-direction with a speed of 3.6×10^6 m/sec. We can measure this speed with a precision of 1%.

(a) With what precision can we simultaneously measure its position? [You may use $\hbar = 1.054 \times 10^{-34}$ J. sec. and $m_e = 9.109 \times 10^{-31}$ kg]

(b) Comment on whether that electron remains within the atom. [Hint: atom to atom distances is typically ~1 Å]

(c) Comment on the positional uncertainty in the y-direction.

[2+1+1= 4 marks]

Q 3. A particle in an infinite square well (width = a) has the initial wave function

$$\psi(x,0) = A\sin^3(\pi x/a) \text{ at } 0 \le x \le a$$

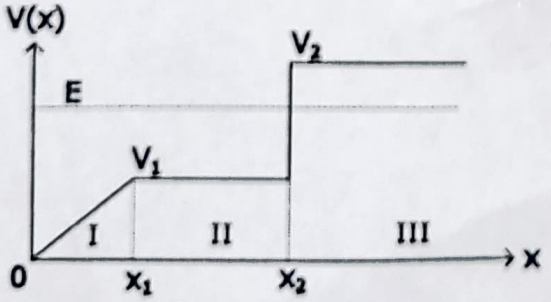
(a) Determine A. [Hint: $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$ and $2\sin\alpha\ \sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$]

(b) Find $\psi(x,t)$.

(c) Find the expected energy value of the particle.

[3+3+2=8 marks]

Q 4. A particle with energy $E < V_2$ but $> V_1$ hits the potential from left, as shown in the figure below.



(Go to next page)

(a) Sketch the real part of eigenfunction (ψ_{real}) for the given potential in regions I, II, and III. Give reasons to justify the nature of your sketch with regards to wavelength and amplitude of ψ_{real}.

(b) Can the particle be stored in region III?

(c) Discuss positional uncertainty Δx qualitatively in three regions. [4+2+2=8 marks]

Q 5. Answer the following questions.

(a) Check whether the following operators are Hermitian: $i(\hat{A} + \hat{A}^{\dagger})$ and $i(\hat{A} - \hat{A}^{\dagger})$.

(b) Consider a ket vector $|\psi\rangle = \sqrt{\frac{5}{8}}|\phi_1\rangle + \sqrt{\frac{2}{8}}|\phi_2\rangle + \sqrt{\frac{1}{8}}|\phi_3\rangle$, where $|\phi_1\rangle$, $|\phi_2\rangle$ and $|\phi_3\rangle$ are a set of orthonormal Eignefunctions of an operator $\hat{A}$. The operator $\hat{A}$ satisfies the Eigenvalue equation $\hat{A}|\phi_n\rangle = (n^2 + 1)|\phi_n\rangle$. Find the expectation value of the operator $\hat{A}$ for the state $|\psi\rangle$.

(c) Let's consider another ket vector $|\chi\rangle = a\,|\phi_1\rangle + \sqrt{\frac{3}{8}}|\phi_2\rangle + \sqrt{\frac{7}{8}}|\phi_3\rangle$ which is orthogonal to above $|\psi\rangle$. Find the value of a.

(d) Determine the probability with which $|\chi\rangle$ collapses to $|\phi_1\rangle$ upon a measurement.

[2+4+2+2=10 marks]

Q 6. A free particle has the initial wave function

$$\psi(x, 0) = Ae^{-ax^2},$$

where A and a are real and positive constants.

(a) Find the value of A. [Hint: $\int_{-\infty}^{\infty} e^{-(ax^2+bx)}dx = \sqrt{\frac{\pi}{a}}e^{b^2/4a}$]

(b) Find the stationary state solution $\psi(x, t)$.

(c) Find the expectation values $\langle x\rangle$ and $\langle p\rangle$.

[2+4+(3+1)=10 marks]