Question from 1-15 contains only one correct choice and carries one mark each. Attempt all questions. Write the correct choices below for question numbers from 1-15.

Question No.	Answer
1	C
2	D
3	C
4	C
5	D
6	B
7	B
8	B
9	A
10	C
11	C
12	A
13	A
14	B
15	D

14

Question from 1-15 contains only one correct choice and carries 1 mark each. Attempt all questions.

1. Due to photo electric effect the electron is ejected from a metal surface having E as its work function. The kinetic energy of the electron ejected have energy E and $2E$ due to the incident light of frequency ν_1 and ν_2 respectively. The ratio ν_1/ν_2 is give by

   (a) 2

   (b) 1/2

   (c) 2/3

   (d) 3/2

2. In a Compton experiment E_1 and E_2 are the energy of the incident and scattered photon respectively. The kinetic energy and momentum of the emitted electron is KE and p respectively, then which of the following statement is correct (m_0 is the rest mass of the electron and c is the velocity of light)

   (a) $KE = E_1 - E_2$

   (b) The momentum of the incident photon is E_1/c.

   (c) The total energy of the electron is $KE + m_o c^2$

   (d) All of the above.

3. The phase velocity of a ripple in water is given by $\sqrt{C\frac{2\pi}{\lambda}}$, where C is a constant for water and λ is the wavelength of the ripple. If the measured phase velocity is $0.5m/s$, then the group velocity is ____m/s

   (a) $\frac{3}{2}$

   (b) $\frac{1}{2}$

   (c) $\frac{3}{4}$

   (d) $\frac{1}{4}$

4. If a particle having energy E, momentum p, de-Broglie wavelength λ and frequency ν, such that $\omega = 2\pi\nu$ and $k = 2\pi/\lambda$, then the velocity of the particle is given by

   (a) $\frac{d\nu}{d\lambda}$

   (b) $\frac{d\omega}{d\lambda}$

   (c) $\frac{dE}{dp}$

   (d) $\frac{dE}{dk}$

   form them

5. For the given functions which one will not be a solution of Schrödinger's equation for all values of x(k is a constant).

   (a) e^{kx}

   (b) $\sin(kx)$

   (c) e^{-kx^2}

   (d) $\sec(kx)$

6. If $\psi(x) = -\iota\sqrt{\frac{1}{2L}}e^{\iota n\pi x/L}$ is a stationary wave function of a particle within a region then the eigenvalue of the momentum operator is ($\iota = \sqrt{-1}, n = 1, 2, 3\cdots$)

   (a) 0

   (b) $\frac{n\pi\hbar}{L}$

   (c) $-\frac{n\pi\hbar}{L}$

   (d) ∞

7. A photon of energy E and $2E$ are emitted from 2 distant star of mass and radius (M, R_1) and $(2M, R_2)$ respectively. Both the photon when it reaches the earth have the same energy $E/2$, then the ratio $\frac{R_1}{R_2} =$

   (a) 1/2

   (b) 3/2

   (c) 2

   (d) 3

8. The result of the following integral $\int_2^6 (3x^2 - 2x - 1)\delta(x - 3)dx$ is

(a) 19

(b) 20

(c) 17

(d) 25

9. If the incidence of ultraviolet light ($\lambda = 350\,\text{nm}$) on a metal leads to emission of photo-electrons with maximum kinetic energy of 1.3 eV from the surface of the metal, the work function of the metal is:

(a) 2.2 eV

(b) 1.3 eV

(c) 1.7 eV

(d) 1.9 eV

10. What is the phase velocity of the de-Broglie wave with frequency 3.5×10^{13} Hz?

(a) 4.32 km/s

(b) 2.30 km/s

(c) 2.57 km/s

(d) 3.17 km/s

11. If the de Broglie wavelength of 100 keV electrons is calculated by considering them as non-relativistic particles, what would be the percentage of error in the calculation (when compared with the relativistic electrons)?

(a) 1.7%

(b) 4.8%

(c) 6.6%

(d) 10.7%

12. In an experimental set-up for photoelectric effect, two light sources of same frequency f but of different intensities I_1 and I_2 are incident on a metal surface. Consider $I_1 = I_2/2$. If the stopping potentials V_{01} and V_{02} are measured between the cathode and the anode corresponding to the two sources, which of the following observation/s is/are true?

(a) $V_{01} = V_{02}$

(b) $V_{01} > V_{02}$

(c) $V_{01} < V_{02}$

(d) $V_{01} = V_{02}/2$

13. What is the expectation value of momentum (in kg·ms^{-1}) of a particle trapped in one-dimensional well?

(a) 0

(b) 0.06

(c) 0.54

(d) 0.77

14. A rigid walled 1-dimensional box that extends from $-L$ to L is divided into three sections by rigid interior walls at $-x$ and x, where $x < L$. Each section contains one particle of same mass in ground states. What is the total energy of the system?

(a) $\frac{\pi^2\hbar}{2m(L-x)^2} + \frac{\pi^2\hbar^2}{2m(2x)^2}$

(b) $\frac{\pi^2\hbar^2}{m(L-x)^2} + \frac{\pi^2\hbar^2}{2m(2x)^2}$

(c) $\frac{2\pi^2\hbar}{m(L-x)^2} + \frac{\pi^2\hbar^2}{2m(2x)^2}$

(d) $\frac{\pi^2\hbar}{m(L-x)^2} + \frac{\pi^2\hbar^2}{2m(2x)^2}$

15. In the case of particle having energy E in a finite potential of height U, which of the following statement is correct, when the particle is inside the potential?

(a) When $E < U$ the wave function is an exponential function in position.

(b) When $U = 0$ the wave function is oscillating function in position.

(c) When $U = E$ the wave function is a linear function in position.

(d) All of the above.

Question from 16-20 carry 7 marks each. Attempt all question. You have to answer within the space given, no extra sheets will be provided.

16. (a) A teacher uses a laser pointer emitting red light for her lectures. The laser pointer has a power output of 5.00 mW. What is the energy of each photon (in Joules)? (hint: wavelength of red light $\lambda_{red} = 650\,\text{nm}$, rest mass of photon is zero, relativistic energy of photons = momentum × vel. of light.) **(2)**

(b) From the above result, find out how many photons are emitted by the laser pointer each second. **(1)**

(c) The work function of sodium is 2.3 eV. What is the maximum wavelength of light that will cause photoelectrons to be emitted from sodium? What will the maximum kinetic energy of the photoelectrons be if light of wavelength 2000 Å falls on a sodium surface? **(4)**

Ans (a) Power $= P = 5\,mW$

$\lambda_{red} = 650\,nm$

2

Energy of photon $= \frac{hc}{\lambda} = \frac{1240}{\lambda(nm)}\,eV = \frac{124\cancel{0}}{65\cancel{0}}\,eV = 1.9\,eV$

(b) $P = \frac{nhc}{\lambda}$ where n is no of photons emitted by the laser per second

1

$5\times10^{-3} = n\times1.9\times\underbrace{1.6\times10^{-19}}_{\text{as energy in Joule take}}$

$1.64\times10^{16} = n$

(c) $\phi = 2.3\,eV$

let λ_{max} = maximum wavelength of light that will cause photoelectrons to be emitted from sodium

$\phi = \frac{hc}{\lambda_{max}}$

$2.3 = \frac{1240}{\lambda_{max}(nm)}$

2

$\lambda_{max} = 539.13\,nm$

If $\lambda = 2000\,\text{Å}$

$E = \frac{124\cancel{00}}{20\cancel{00}} = 6.2\,eV$

$$E = KE_{max} + \phi$$

$$6.2 = KE_{max} + 2.3$$

$$KE_{max} = 3.9$$

17. Suppose we have a potential barrier spread between $-a$ to a of height U and every where else $U = 0$. If the mass is m and energy of the tunneling particle is equal to U?

(a) Write down the general solution of the wave function in all three region. (3

(b) Calculate the transition probability of the particle ? (4

$U\psi$ $$-\frac{\hbar^2}{2m}\frac{\delta^2\psi}{\delta x^2} = E\psi$$

$$\frac{\delta^2\psi}{\delta x^2} + \frac{2m}{\hbar^2}(E-U)\psi = 0$$

for region $x < -a$

$$\psi = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi(x+a)}{L}\right)$$

for region $x > a$

$$\psi = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi(x-a)}{L}\right)$$

0

18. A photon having rest mass energy of the electron undergoes Compton scattering. The photon is scattered at angle $\phi = 62°$ with respect to the incoming photon direction.

(a) Calculate the energy (in Joules) of the scattered photon and electron. (4)

(b) Calculate the angle at which the electron scatters with respect to the incoming photon direction? (3)

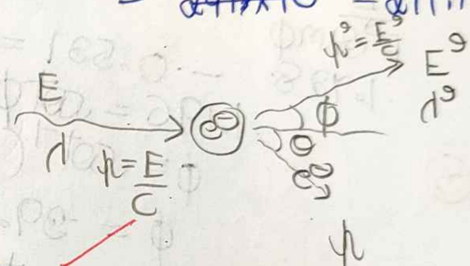
(a) Energy of photon = rest mass energy of electron

$E = m_0c^2 = 81.9 \times 10^{-15}$ J

$\frac{hc}{\lambda} = m_0c^2 \Rightarrow \lambda = \frac{h}{m_0c} = \frac{6.62 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^8}$

$= 0.241\cancel{3} \times 10^{-11}$

$= \cancel{241.7 \times 10^{-14}} = 241.75 \times 10^{-14}$



$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\phi)$

$\lambda' - \frac{h}{mc} = \frac{h}{mc} - \frac{h}{mc}\cos(62°)$

$\lambda' = \frac{2h}{mc} - \frac{h}{mc}(0.47)$

$\lambda' = \frac{h}{mc}(2 - 0.47) = 370 \times 10^{-14}$ m

Energy of scattered photon $= \frac{hc}{\lambda'} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{370 \times 10^{-14}}$

$= 0.0536 \times 10^{-12}$

$E' = 53.67 \times 10^{-\cancel{12}15}$ J

Kinetic Energy of electron $= E - E' = 81.9 \times 10^{-15} - 53.67 \times 10^{-15}$

$= 28.23 \times 10^{-15}$ J

Total energy of electron $= 28.23 \times 10^{-15} + m_0c^2$

$= 28.23 \times 10^{-15} + 81.9 \times 10^{-15}$

$= 110.13 \times 10^{-15}$

(b)

Momentum conservation along x-axis

$\frac{E}{c} = p\cos\theta + \frac{E'}{c}\cos\phi$ — (1)

Momentum conservation along y-axis

$$\frac{E'}{c}\sin\phi = p\sin\theta$$

$$\frac{E'}{c}\frac{\sin\phi}{\sin\theta} = p \quad -(2)$$

Put in eqn (1)

$$\frac{E}{c} = \frac{E'\sin\phi}{c\sin\theta}\cos\theta + \frac{E'}{c}\cos\phi$$

$$\frac{E}{c} - \frac{E'}{c}\cos\phi = \frac{E'}{c}\sin\phi\cot\theta$$

$$\frac{E}{E'\sin\phi} - \frac{E'\cos\phi}{E'\sin\phi} = \cot\theta$$

$$\frac{E}{E'\sin\phi} - \cot\phi = \cot\phi$$

$$1.728 - 0.531 = \cot\phi$$

$$1.196 = \cot\phi$$

(3)

$$\phi = \cot^{-1}(1.196)$$

$$\phi = 39.89^\circ \approx 40^\circ$$

So electron scatters at an angle of 39.89° with respect to incoming photon direction

19. (a) If the wave function representing a particle of mass m moving freely inside an infinite potential well of length L at time $t = 0$ is $\phi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$, determine the expectation value of the kinetic energy T_n corresponding to the nth state. (2)

(b) A quantum state is given by $\psi(x) = \frac{1}{\sqrt{3}}\phi_1(x) + \frac{2}{3}\phi_2(x) + \frac{\sqrt{2}}{3}\phi_3(x)$, where ϕ's satisfy the condition

$$\int \phi_m^*(x)\phi_n(x)dx = \begin{cases} 0, & m \neq n \\ 1, & m = n \end{cases}$$

If $P_n = \left|\frac{\int \phi_n^*(x)\psi(x)dx}{\int \psi^*(x)\psi(x)dx}\right|^2$ is the probability of the system being in the state $\phi_n(x)$, show that $P_2 > P_1 > P_3$. (5)

Q19)

a)

Ans(a) $\phi_n(x) = \frac{\sqrt{2}}{\sqrt{L}} \sin\left(\frac{n\pi x}{L}\right)$

$$\frac{\sqrt{2mE}}{\hbar} = \frac{n\pi}{L}$$

$$\phi_n(x,t) = \frac{\sqrt{2}}{\sqrt{L}} \sin\left(\frac{n\pi x}{L}\right) e^{\frac{-iEt}{\hbar}}$$

$$\frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{L^2}$$

$$E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

Expectation value of kinetic energy $= T_n = \int_{-\infty}^{\infty} \psi^* \hat{E} \psi \, dx$

$$= \int_{-\infty}^{\infty} \frac{\sqrt{2}}{\sqrt{L}} \sin\left(\frac{n\pi x}{L}\right) e^{\frac{iEt}{\hbar}} \frac{\hbar}{i} \frac{\delta}{\delta t} \frac{\sqrt{2}}{\sqrt{L}} \sin\left(\frac{n\pi x}{L}\right) e^{\frac{-iEt}{\hbar}} dx$$

$$T_n = \int_{-\infty}^{\infty} \frac{2}{L} \sin^2\left(\frac{n\pi x}{L}\right) \cancel{e^{\frac{iEt}{\hbar}}} \frac{\cancel{\hbar}}{\cancel{i}} \left(\frac{-\cancel{i}E}{\cancel{\hbar}}\right) \cancel{e^{\frac{-iEt}{\hbar}}} dx$$

$$= \int_{-\infty}^{\infty} -\frac{2E}{L} \sin^2\left(\frac{n\pi x}{L}\right) dx$$

b)

Ans b) $\psi(x) = \frac{1}{\sqrt{3}}\phi_1(x) + \frac{2}{3}\phi_2(x) + \frac{\sqrt{2}}{3}\phi_3(x)$

$$P_1 = \frac{\left|\int \phi_1^*(x)\psi(x)dx\right|^2}{\int \psi^*(x)\psi(x)dx} \qquad P_2 = \frac{\left|\int \phi_2^*(x)\psi(x)dx\right|^2}{\int \psi^*(x)\psi(x)dx}$$

$$P_3 = \frac{\left|\int \phi_3^*(x)\psi(x)dx\right|^2}{\int \psi^*(x)\psi(x)dx}$$

$$\psi^*(x) = \frac{1}{\sqrt{3}}\phi_1^*(x) + \frac{2}{3}\phi_2^*(x) + \frac{\sqrt{2}}{3}\phi_3^*(x)$$

$$\int \psi^*(x)\psi(x)dx = \int \left[\frac{1}{\sqrt{3}}\phi_1^*(x) + \frac{2}{3}\phi_2^*(x) + \frac{\sqrt{2}}{3}\phi_3^*(x)\right]\left[\frac{1}{\sqrt{3}}\phi_1(x) + \frac{2}{3}\phi_2(x) + \frac{\sqrt{2}}{3}\phi_3(x)\right]$$

By using $\int \phi^*_m(x)\, \phi^*_n(x)\, dx = 0 \quad m \neq n$
$\qquad\qquad\qquad\qquad\qquad\quad = 1 \quad m = n$

$$= \frac{1}{3}\int \phi_1^*(x)\,\phi_1(x)\,dx + \frac{4}{9}\int \phi_2^*(x)\phi_2(x)\,dx + \frac{2}{9}\int \phi_3^*(x)\phi_3(x)\,dx$$

$$\int \psi^*(x)\,\psi(x)\,dx = \frac{1}{3}+\frac{4}{9}+\frac{2}{9} = 1$$

$$P_1 = \frac{\left|\int \phi_1^*(x)\,\psi(x)\,dx\right|^2}{\int \psi^*(x)\psi(x)\,dx} = \frac{\left|\int \phi_1^*(x)\frac{1}{\sqrt{3}}\,\phi_1(x)\,dx\right|^2}{1}$$

→ where other terms zero by this condition

$$P_1 = \frac{1}{3}$$

$$P_2 = \frac{\left|\int \phi_2^*(x)\,\psi(x)\,dx\right|^2}{\int \psi^*(x)\,\psi(x)\,dx} = \frac{\left|\int \phi_2^*(x)\frac{2}{3}\,\phi_2(x)\,dx\right|^2}{1}$$

$$P_2 = \frac{4}{9}$$

$$P_3 = \frac{\left|\int \phi_3^*(x)\,\psi(x)\,dx\right|^2}{\int \psi^*(x)\psi(x)\,dx} = \frac{\left|\int \phi_3^*(x)\frac{\sqrt{2}}{3}\,\phi_3(x)\,dx\right|^2}{1}$$

$$P_3 = \frac{2}{9}$$

We can clearly see that

$$\frac{4}{9} > \frac{1}{3} > \frac{2}{9}$$

so $P_2 > P_1 > P_3$

5

20. A particle is represented (at time $t = 0$) by the wave function

$$\psi(x) = \begin{cases} \sqrt{15/(16a^5)}(a^2 - x^2), & -a \le x \le a \\ 0, & otherwise \end{cases}$$

(a) Determine the uncertainty in position $\sigma = \langle x^2 \rangle - (\langle x \rangle)^2$. (4)

(b) Find the expectation value of momentum. (3)

$$\langle x \rangle = \int_{-\infty}^{\infty} x |\psi|^2 dx = \int_{-a}^{a} x \left(\frac{15}{16a^5}\right)(a^2-x^2)^2 dx$$

$$= \int_{-a}^{a} \frac{15}{16a^5} x (a^4 + x^4 - 2x^2a^2) dx$$

$$= \frac{15}{16a^5} \int_{-a}^{a} (a^4x + x^5 - 2x^3a^2) dx$$

$$= \frac{15}{16a^5} \left[\frac{a^4x^2}{2} + \frac{x^6}{6} - \frac{2x^4a^2}{4}\right]_{-a}^{a}$$

$$\frac{15}{16a^5}\left[\frac{a^4a^2}{2} + \frac{a^6}{6} - \frac{2a^6}{4} - \left(\frac{a^4a^2}{2} + \frac{a^6}{6} - \frac{2a^6}{4}\right)\right]$$

$$\langle x \rangle = 0$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 |\psi|^2 dx = \int_{-a}^{a} x^2 \left(\frac{15}{16a^5}\right)(a^2-x^2)^2 dx$$

$$= \int_{-a}^{a} \frac{15}{16a^5} x^2 [a^4 + x^4 - 2a^2x^2] dx$$

$$= \int_{-a}^{a} \frac{15}{16a^5} [a^4x^2 + x^6 - 2a^2x^4] dx$$

→ it is even function

$$= \frac{15 \times 2}{16a^5} \int_{0}^{a} (a^4x^2 + x^6 - 2a^2x^4) dx$$

$$= \frac{15}{8a^5} \left[\frac{a^4x^3}{3} + \frac{x^7}{7} - \frac{2a^2x^5}{5}\right]_0^a$$

$$\frac{15}{8a^5}\left[\frac{a^4a^3}{3} + \frac{a^7}{7} - \frac{2a^7}{5} - 0\right]$$

$$\frac{15}{8a^5}\left[\frac{a^7}{3} + \frac{a^7}{7} - \frac{2a^7}{5}\right]$$

$$= \frac{\cancel{15}^{3}}{\cancel{8}a^5}\left(\frac{\cancel{8}a^7}{\cancel{105}_{21\ 7}}\right)$$

$$\langle x^2 \rangle = \frac{a^2}{7}$$

$$\sigma = \langle x^2 \rangle - (\langle x \rangle)^2$$

$$\sigma = \frac{a^2}{7} - 0$$

$$\boxed{\sigma = \frac{a^2}{7}}$$

Expectation value of momentum $= \int_{-\infty}^{\infty} \psi^* \hat{p} \psi \, dx$

$$= \int_{-a}^{a} \sqrt{\frac{15}{16a^5}} \frac{\hbar}{i} \frac{\delta}{\delta x} (a^2 - x^2)\left(\frac{\hbar}{i}\frac{\delta}{\delta x}\psi\right) dx$$

$$= \int_{-a}^{a} \sqrt{\frac{15}{16a^5}} (a^2 - x^2) \frac{\hbar}{i} \frac{\delta}{\delta x} \left(\frac{\sqrt{15}}{\sqrt{16a^5}} (a^2 - x^2)\right) dx$$

$$\frac{\hbar}{i} \frac{15}{16a^5} \int_{-a}^{a} (a^2 - x^2)(-2x) \, dx$$

$$-2 \times \frac{\hbar}{i} \frac{15}{16a^5} \int_{-a}^{a} (a^2 x - x^3) \, dx$$

↳ As it is odd function so it's integral is zero

Expectation value of momentum $= 0$