Minor Exam

PYL101: Electromagnetics and Quantum Mechanics

10 AM – 12 Noon, Dec 18, 2022

Name: Entry No.: Group No.:

Instructions

1. Write clearly on the front page of your answer sheet, your name, entry number, group number, and course code.
2. In the question paper, there are 12 problems over 2 pages, followed by formulae sheet for your use, if required.
3. All problems are compulsory and marks for each part are indicated.
4. Use of mobile phone is strictly prohibited. However, you're allowed the use a non-graphing scientific calculator.
5. Use of unfair means or abetment gets you a straight zero and all such cases will be forwarded to the disciplinary committee.

Problem 1

Verify Stokes' theorem for $\mathbf{A} = (2x - y)\hat{\mathbf{x}} - 2yz^2\hat{\mathbf{y}} - 2zy^2\hat{\mathbf{z}}$ on the upper hemispherical surface of a sphere of radius 2 centered at the origin. *[4 marks]*

Problem 2

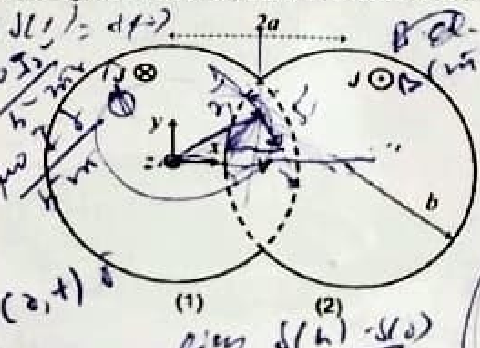
2.1. Investigate *whether or not* a field distribution of the form $\mathbf{F} = f(r,t)\hat{\mathbf{r}}$ (where all the symbols have their usual meaning in spherical coordinates, and t is time) admits a scalar potential ψ. *[2 marks]*

2.2. For the one-dimensional Dirac delta function, $\delta(x)$, show the following *[2 marks]*

$$\delta'(x) = -\frac{\delta(x)}{x}$$

Problem 3

Long copper wires (1) and (2), carrying opposing uniform currents of magnitude J, have a cross section given by two intersecting circles of radius b with centers separated by $2a$. However, note that the *lens-shaped* region V bounded by the dotted lines is vacuum. Given the coordinate system below, find $\mathbf{B}$ inside the region V. *[4 marks]*



Problem 4

The magnetic field $\mathbf{B}$ inside a piece of iron, modelled as a linear magnetic material with $\mu_{iron} = 1000\mu_0$ is directed at 85 off the normal to the iron-air interface defined by $z = 0$.

4.1. Calculate the direction (in degrees °) of the auxiliary field $\mathbf{H}$ in air with respect to the normal. *[2 marks]*

4.2. If the magnetic field B in iron is $1\ T$, what is its magnitude in air? *[2 marks]*

Problem 5

A conducting rod of length l rotates about z-axis passing through one end of the rod with angular velocity of rotation, ω the region of uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$.

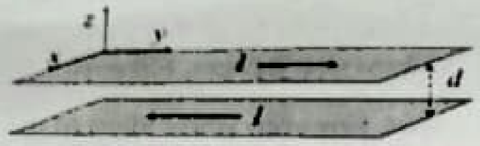
5.1. Find the *emf* between the two ends of the rod. *[2 marks]*

5.2. What is the *emf* between the fixed end and the center of the rod. *[2 marks]*

Problem 6

Two conducting ribbons carry current I in opposite directions, as shown in the figure below, by maintaining a potential difference of V between them. Uniformly distributed surface charge density, σ and current density, $\mathbf{K}$ are given and the gap between the ribbons is d. By calculating the Poynting vector, find the magnitude and direction of electromagnetic energy transported per unit time per unit area. Compare this result with what is obtained directly in terms of V.

[3 + 1 marks]



Problem 7

An electromagnetic wave that is propagating through a linear dielectric medium, has its electric field component expressed in complex form as $\mathbf{E} = 10^4(2\hat{\mathbf{x}} - \hat{\mathbf{y}})e^{i10^7(x+2y+2z-1.5\times10^8 t)}$. Here, different quantities are in mks system of units. Find the following properties.

7.1. Direction of polarization of the EM wave. *[1 mark]*
7.2. Direction of propagation of the EM wave. *[1 mark]*
7.3. The frequency and the magnitude of the wavevector in the medium. *[1 mark]*
7.4. The real vector magnetic field component of the wave. *[1 mark]*
7.5. The refractive index of the medium. *[1 mark]*

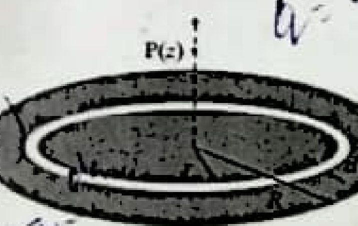
Problem 8

Suppose a plane wave of angular frequency ω, propagating in the z-direction and polarized in the x-direction, is normally incident onto an interface between two linear dielectric media ($\varepsilon_1 \neq \varepsilon_2, \mu_1 \neq \mu_2$).

8.1. Prove that the polarization of the reflected as well as the transmitted waves remains as that of the incident wave. *[2 marks]*
8.2. Calculate the reflection and transmission coefficients of the wave at the interface. *[2 marks]*

Problem 9

A flat circular disc, of radius R, carries a uniform surface charge density σ. Find the electric field a distance z above the center of the final structure if a ring of radius $r < R$, and width d ($<< r$), has been removed as shown in the figure below. *[3 marks]*



Problem 10

An inverted hemispherical bowl of radius R carries a uniform surface charge density σ. Find the potential difference between the north pole and the center. *[3 marks]*

Problem 11

If the electric susceptibility in the entire region is given as $\chi_e = \chi_0[1 - H(z)]$, where $H(z)$ is the standard Heaviside step function and χ_0 is a constant, and there is a point charge q situated at $(0,0,d)$. Calculate the surface bound charge density and the total bound charge. *[3 marks]*

Problem 12

A long cylinder of radius, R, is filled with material of dielectric constant, ϵ_r and embedded free charge with density, $\rho = ks$, s being the distance from the axis and k, a constant. Calculate the total energy stored and separate the contributions due to the electrostatic energy, and the internal energy due to polarization. *[3 marks]*