Name, Entry No. and Group No.: [redacted]

1 Multiple-choice questions

[1 × 26 = 26 marks]

The alphabetical letter (A,B,C or D) corresponding to your answer should be written clearly within the square brackets below. Any ambiguity or untidiness will be awarded zero points. All symbols have their usual meanings unless otherwise indicated. There is no negative marking.

Answers

✓	1. [B]		14. [B]	✗	
✓	2. [A]		15. [C]	✓	
✓	3. [D]		16. [C]	✓	
✓	4. [B]		17. [A]	✓	
✓	5. [B]		18. [A]	✗	✓
✗	6. [C]		19. [B]	✗	
✓	7. [A]		20. [B]	✗	
✓	8. [A]		21. [D]	✓	
✗	9. [C]		22. [B]	✓	
✓	10. [C]		23. [C]	✓	
✓	11. [A]		24. [B]	✗	
✗	12. [B]		25. [B]	✗	
✗	13. [A]		26. [B]	✓	

TOTAL: 16 + 2

1. The derivative of the Dirac delta function $\delta(x)$ is

A. $\frac{\delta(x)}{x}$

B. $-\frac{\delta(x)}{x}$

C. $\frac{\delta(x)}{x^2}$

D. $\frac{2\delta(x)}{x^2}$

2. $\int_B zr\sin\theta d\tau$ where B is a cylindrical box described by $B = \{0 \le r \le 2, 0 \le \theta \le \pi/2, 0 \le z \le 4\}$ is

A. $\frac{64}{3}$

B. $\frac{32}{3}$

C. $\frac{16}{3}$

D. $\frac{128}{3}$

3. The rate of change of the scalar field $f(x, y, z) = xy + 2z^2$ at $(1, 1, 1)$ in the direction of the vector $\hat{x} - 2\hat{y} + \hat{z}$ is

A. $\frac{4}{\sqrt{6}}$

B. $\frac{5}{\sqrt{6}}$

C. $\frac{7}{\sqrt{6}}$

D. $\frac{3}{\sqrt{6}}$

4. The line integral of the vector field $\boldsymbol{F} = \hat{x} + 2\hat{y} + \hat{z}$ along a circular arc of unit radius from $(1, 0, 1)$ to $(0, 1, 1)$ is

A. 2

B. 1

C. 1/2

D. 1/3

5. A vector field in spherical coordinates is

$$\boldsymbol{E} = \begin{cases} kr\hat{r} & r \le a \\ k\frac{a^3}{r^2}\hat{r} & r > a \end{cases}$$

where k and a are constants. The divergence of this field is

A. $\begin{cases} 2k & r \le a \\ 0 & r > a \end{cases}$

B. $\begin{cases} 3k & r \le a \\ 0 & r > a \end{cases}$

C. $\begin{cases} k & r \le a \\ 0 & r > a \end{cases}$

D. $\begin{cases} 2k & r \le a \\ \infty & r > a \end{cases}$

6. For the problem above, the outward flux through the sphere $r = b \le a$ is

A. $2\pi kb^3$

B. $8\pi kb^3$

C. πkb^3

D. $4\pi kb^3$

7. The electric field $\boldsymbol{E}$ at the surface of a conductor is known to be perpendicular, pointing outward, equal to 100 V/m, and the conductor is in free space. The charge density on its surface is

A. 8.854×10^{-10} C/m^2

B. 4.427×10^{-10} C/m^2

C. -4.427×10^{-10} C/m^2

D. -8.854×10^{-10} C/m^2

8. The self capacitance of the Earth assuming it's a conducting sphere of radius 6400 km is

A. 7.12×10^{-4} F

B. 7.12 mF

C. 7.12×10^4 F

D. 7.12 MF

9. For a cylindrical bar electret with a uniform $\boldsymbol{P}$ along its axis, the D-field lines

A. form open and closed loops

B. only form closed loops

C. only form open loops

D. are uniformly zero

10. For a uniformly polarized spherical electret surrounded by vacuum. Outside it, in the far field region

A. $\boldsymbol{E} = 0, \boldsymbol{D} \neq 0$

B. $\boldsymbol{E} \neq 0, \boldsymbol{D} = 0$

C. $\boldsymbol{E} \neq 0, \boldsymbol{D} \neq 0$

D. $\boldsymbol{E} = 0, \boldsymbol{D} = 0$

11. The energy density of an electric field $\boldsymbol{E}$ within an infinite, homogenous, and isotropic medium of permittivity ϵ is

A. $\frac{\epsilon E^2}{2}$

B. $\frac{\epsilon^2 E^2}{2}$

C. $\frac{3\epsilon^2 E^2}{2}$

D. $\frac{\epsilon^2 E^2}{3}$

12. The *local* conservation of electric charge demands that

A. $\nabla \cdot \boldsymbol{J} = +\frac{\partial \rho}{\partial t}$

B. $\nabla \cdot \boldsymbol{J} = +\frac{d\rho}{dt}$

C. $\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t}$

D. $\nabla \cdot \boldsymbol{J} = +\frac{d\rho}{dt}$

13. A straight dimagnetic rod of susceptibility χ and radius 'a' carries a uniformly distributed current I. In terms of azimuthal angle ϕ, the magnetization at a perpendicular distance 's' ($s < a$) from the axis of the rod is given as

A. $|\chi|\frac{I}{2\pi a^2}s\hat{\phi}$

B. $\chi\frac{I}{2\pi a^2}s\hat{\phi}$

C. $-\chi\frac{I}{2\pi a^2}s\hat{\phi}$

D. 0

14. Let $A_1 = \frac{1}{2}(xB\hat{y} - yB\hat{x})$ and $A_2 = \frac{1}{2}rB\sin\theta\hat{\phi}$ be two vector potentials. B is a constant and $r = \sqrt{x^2 + y^2 + z^2}$. Let $\boldsymbol{B_1}$ and $\boldsymbol{B_2}$ be the corresponding magnetic fields. Then

A. $|\boldsymbol{B_1}| > |\boldsymbol{B_2}|$

B. $|\boldsymbol{B_1}| = |\boldsymbol{B_2}|$ but $\boldsymbol{B_1} \neq \boldsymbol{B_2}$

C. $B_1 = B_2$

D. $|B_1| < |B_2|$

15. Consider 4 vector fields.

$$\begin{aligned} E_1 &= (xy)\hat{x} + (2yz)\hat{y} + (3xz)\hat{z} \\ E_2 &= (y^2)\hat{x} + (2xy + z^2)\hat{y} + (2yz)\hat{z} \\ E_3 &= (x^2z)\hat{x} - (2xz)\hat{y} + (yz)\hat{z} \\ E_4 &= (xz^3)\hat{x} - (2x^2yz)\hat{y} + (2yz^4)\hat{z} \end{aligned}$$

Which of the vector field is conservative

A. E_1, E_2, E_3

B. E_2, E_4

C. E_2

D. E_1, E_3

16. For an object with a uniform Magnetization M and no free currents, which of the following is true

A. Surface current density is uniform

B. Volume current density is uniform

C. Volume current density is zero

D. Surface current density is zero

17. Consider an Electric potential in the form of a mountain. The Electric field points in general:

A. Down the mountain sides

B. Up the Mountain sides

C. Around the mountain side

D. Perpendicular to the sides of the mountain.

18. Two points charges of equal charge q are kept at points x=10 and x=-5. Assuming the positive normal for the $Y-Z$ plane to be in the $\hat{i}$ direction, the net flux through the $Y-Z$ plane is

A. Zero

B. Negative

C. Positive

D. Not enough information

19. Consider a surface carrying an current density K. It is found that the perpendicular component of the Auxillary field H, changes by an amount of 2 units on moving across the surface. Which of the following is true

A. The perpendicular component of magnetization changes by 2 units

B. The perpendicular component of magnetization changes by 2 μ_0 units where μ_0 is the permeability of free space

C. The perpendicular component of magnetizaation changes by -2 units

D. The perpendicular component of magnetization does not change

20. The Electric field in a linear dielectric material (permitivity ϵ) is found to be $E = \frac{A\hat{r} + B\sin\theta\cos\phi\hat{\phi}}{r}$. A and B are constants. The Volume charge density is

A. $\rho = 0$

B. $\rho = \epsilon\frac{A}{r^2} - \epsilon\frac{B}{r^2}\sin\phi$

C. $\rho = \epsilon_0\frac{A}{r^2} - \epsilon_0\frac{B}{r^2}\sin\phi$

D. $\rho = \epsilon_0\frac{A}{r^2} + \epsilon_0\frac{B}{r^2}\sin\phi$

21. An infinitely long cylinder of radius 'r' and its axis along the z axis, is wrapped by a wire. The current in the wire is zero. The material has a frozen in magnetization $M = \alpha s\hat{z}$ where 's' is the distance from the axis and α is a constant. Which of the following is true

A. $\nabla \times H = 0$

B. $B = \mu_0\alpha s\hat{z}$

C. $\nabla . H = 0$

D. All of the above

22. Consider a conductor with a surface charge distribution σ. If the surface charge is doubled, then the force per unit area on the surface

A. Doubles

B. Becomes 4 times

C. Unchanged

D. Becomes half

23. Consider a surface charge distribution. The perpendicular component of the electric field changes by an amount of 2 units between 2 points separated by ϵ :, $\epsilon \to 0$, but on either end of the surface. The potential between these two points

A. Increases by 2 units

B. Decreases by 2 units

C. Unchanged

D. Halved

24. Consider two point charge of charge 'q' and '-q' kept at coordinates $(-1, 1, 1)$ and $(1, 1.2)$ respectively. The length units on each axis are in cm. The total flux of the electric displacement vector through a sphere of radius 2 cm and centered at origin is

A. 0

B. q/ϵ_0

C. q

D. $2q$

25. Consider the following statements:
Statement A: $\nabla . E = 0$ in vacuum
Statement B: $\nabla \times B = 0$ in vacuum
Statement C: $\nabla \times P \neq 0$ inside a homogenous linear dielectric material

A. Only A and B are true

B. Only A and C are true

C. Only B and C are true

D. A,B and C are true

26. Consider a capacitor made of two concentric spherical metal shells of radii α and β, $\beta > \alpha$. If the radius of each is doubled then

A. Capacitance increases by a factor of 4

B. Capacitance increases by a factor of 2

C. Capacitance is unchanged

D. Capacitance reduces by a factor of 2

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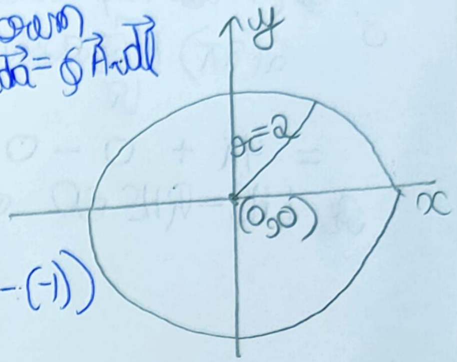
2 Descriptive questions

[4 × 6 = 24 marks]

1. Verify Stokes' theorem for the vector field $A = (2x-y)\hat{x} - 2yz^2\hat{y} - 2zy^2\hat{z}$ on the upper hemispherical surface of a sphere of radius 2 centered at the origin (above the $x-y$ plane), where C is its boundary (rim of the surface in the $x-y$ plane). [6 marks]

$\vec{A} = (2x-y)\hat{x} - 2yz^2\hat{y} - 2zy^2\hat{z}$ Stokes's theorem

$\int(\nabla\times\vec{A})\cdot d\vec{a} = \oint \vec{A}\cdot d\vec{l}$



$$\nabla\times\vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\delta}{\delta x} & \frac{\delta}{\delta y} & \frac{\delta}{\delta z} \\ (2x-y) & -2yz^2 & -2zy^2 \end{vmatrix}$$

$= \hat{x}(-4yz-(-4yz)) - \hat{y}(0-0) + \hat{z}(0-(-1))$

$\hat{x}(-4yz+4yz) - \cancel{0} + \hat{z}$

$= \hat{z}$

As it would be very tough to do by cartesian coordinates so we convert it into spherical coordinates

here $r=2$, θ vary from 0 to $\frac{\pi}{2}$, ϕ vary from 0 to 2π

so $\nabla\times\vec{A} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

$d\vec{a} = r^2\sin\theta\,\hat{r}\,d\theta\,d\phi$

LHS $= \int(\nabla\times\vec{A})\cdot d\vec{a} = \int(\cos\theta\,\hat{r} - \sin\theta\,\hat{\theta})\cdot(r^2\sin\theta)\hat{r} = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} r^2\cos\theta\sin\theta\,d\theta\,d\phi$

$= \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \frac{4\sin 2\theta}{2}\,d\theta\,d\phi$

$= \int_0^{2\pi} d\phi \int_0^{\frac{\pi}{2}} 2\sin 2\theta\,d\theta$

$= 2\pi\times 2\left[-\frac{\cos 2\theta}{2}\right]_0^{\frac{\pi}{2}}$

$= 2\pi\left[\cancel{-(-1)+1}\right]\left[-(-1)+1\right]$

LHS $= \cancel{2\pi}\ 4\pi$

RHS

Using spherical coordinates

$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$

For $\oint\vec{A}\cdot d\vec{l}$ around C

$r=2$, $dr=0$, $\theta=\frac{\pi}{2}$, $d\theta=0$, & ϕ vary from 0 to 2π

$z = r\cos\theta = 2\cos\frac{\pi}{2} = 0$

so $d\vec{l} = 2\,d\phi\,\hat{\phi}$

so we only need term of $\hat{\phi}$ in $\vec{A}$ so

$A_\phi = (2r\sin\theta\cos\phi - r\sin\theta\sin\phi)(-\sin\phi\,\hat{\phi}) - 0 - 0$

$(2\times 2\sin\frac{\pi}{2}\cos\phi - 2\sin\frac{\pi}{2}\sin\phi)(-\sin\phi\,\hat{\phi}) = (4\cos\phi - 2\sin\phi)(-\sin\phi\,\hat{\phi})$

$= -2\sin 2\phi + 2\sin^2\phi$

$$= -2\sin 2\phi + 2(1-\cos 2\phi)$$

$$= 1 - 2\sin 2\phi - \cos 2\phi$$

$$\int_0^{2\pi} (1-2\sin 2\phi - \cos 2\phi)\, 2\, d\phi$$

$$2(2\pi) + \left[\frac{2}{2}\cos 2\phi - \frac{\sin 2\phi}{2}\right]_0^{2\pi}$$

$$= 4\pi + 0 - 0$$

∴ LHS = RHS so Stokes law is verified

6

6

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2. A very long cylindrical ideal conductor of $r = a$ [m] (connected to zero potential) is covered with a material of dielectric constant ϵ out to $r = b > a$ [m]. A uniform volume charge density ρ_0 [C/m^2] is distributed throughout the dielectric. [6 marks]

   (a) (2 points) Calculate the potential everywhere inside the charge layer.

   (b) (2 points) Calculate the potential outside the charge layer.

   (c) (2 points) Plot the potential in space.

3. A sphere of radius 'R' has a volume charge distribution $\rho = \alpha r^2$, where α is a constant and 'r' is radial distance from the centre. [6 marks]

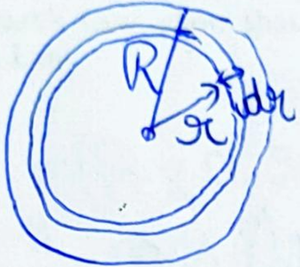
(a) (4 points) Find the energy of the configuration.

(b) (2 points) Using your result, estimate the change in the energy if the radius of the cylinder is halved.

sphere

Q3

Ans 3



$\rho = \alpha r^2$

$$Q = \int_0^R \alpha r^2 \, 4\pi r^2 \, dr$$

$$Q_{total} = \int_0^R 4\pi \alpha r^4 \, dr$$

$$Q_{Total} = \frac{4\pi\alpha R^5}{5}$$

+1

4. Work out the following questions: **[6 marks]**

(a) (1 points points) Let $\boldsymbol{B}$ be a uniform magnetic field. The corresponding vector potential is $\boldsymbol{A}(\boldsymbol{r}) = (\boldsymbol{B} \times \alpha \boldsymbol{r})$. Find α and what are its units.

(b) (2 points points) A uniform surface current flows on the $X - Y$ plane with a surface current density $J\hat{\boldsymbol{x}}$ where J is a number. Find the Auxillary field $\boldsymbol{H}$ at every point in space.

(c) (3 points points) Consider two arbitrary shaped closed loops C_1 and C_2 carrying current I_1 and I_2 respectively. Using Biot Savart's Law, show that the force due to the magnetic field due to one loop on the other follows the Newton's third Law.

6 a)