

4. (i) A plasma with an isotropic velocity distribution is placed in a magnetic mirror trap with mirror ratio $R_m = 4$. There are no collisions, so the particles in the loss cone simply escape, and the rest remain trapped. What fraction is trapped?

(ii) Consider a rectangular wave guide with dimensions $2.28 \text{ cm} \times 1.01 \text{ cm}$. What TE modes will propagate in this wave guide, if the driving frequency is $1.70 \times 10^{10} \text{ Hz}$? Suppose you wanted to excite only one TE mode, what range of frequencies could you use? What are the corresponding wavelengths (in open space)?

(iii) Write down the (real) electric and magnetic fields for a monochromatic plane wave of amplitude E_0 , frequency ω and phase angle zero that is (a) travelling in the negative x direction and polarized in the z direction; (b) travelling in that direction from the origin to the point $(1, 1, 1)$, with polarization parallel to the xy plane.

(3+4+4)

Constants:

$$e = 1.60 \times 10^{-19} \text{ C} \quad m_e = 9.11 \times 10^{-31} \text{ Kg} \quad \epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$\mu_0 = 4 \pi \times 10^{-7} \text{ H/m} \quad c = 3.0 \times 10^8 \text{ m/s}$$

Lienard-Wiechert potentials for a moving point charge:

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0} \frac{qc}{(rc - \mathbf{a} \cdot \mathbf{v})}, \quad \mathbf{A}(\mathbf{r}, t) = \frac{\mu_0}{4\pi} \frac{q\mathbf{v}}{(rc - \mathbf{a} \cdot \mathbf{v})}$$

Fields of a moving point charge:

$$\mathbf{E}(\mathbf{r}, t) = \frac{q}{4\pi\epsilon_0} \frac{\mathbf{a}}{(\mathbf{a} \cdot \mathbf{u})^3} [(c^2 - v^2)\mathbf{u} + \mathbf{a} \times (\mathbf{u} \times \mathbf{a})], \quad \mathbf{B}(\mathbf{r}, t) = \frac{1}{c} \hat{\mathbf{a}} \times \mathbf{E}(\mathbf{r}, t)$$

where the symbols have their usual meanings

$$\int_0^\pi \sin^3 \theta d\theta = \frac{4}{3}$$

Indian Institute of Technology, Delhi

Department of Physics

EPL208 Electrodynamics and Plasmas

Second Semester 2013-2014

Major

Duration: 2 hour

Marks: 40

2014

Date: 05 May

1. (a) Suppose $V = 0$ and $A = A_0 \sin(kx - \omega t)$, where A_0 , ω and k are constants. Find E and B , and check that they satisfy Maxwell's equations in vacuum. What condition must you impose on ω and k ?

(b) A particle of charge q moves in a circle of radius a at constant angular velocity ω . Assume that the circle lies in the xy plane, centered at the origin, and at time $t = 0$ the charge is at $(a, 0)$, on the positive x axis. Find the Lienard-Wiechert potentials for points on the z axis. (4+4)

(i) In the case of oblique incidence of em wave on a boundary between two linear media, show that the angle of reflection is equal to angle of incidence and the angle of refraction follows the well known Snell's law.

(ii) Consider a Hertzian dipole an infinitesimal current element $I dl$ located at the origin of co-ordinate system such that dl is oriented in z direction and carries a current $I(t) = I_0 \cos \omega t$. Calculate the retarded potential A and then the B and E fields. Further show that the power radiated by the dipole is given by

$$P_{rad} = 40\pi^2 \left[\frac{dl}{\lambda} \right]^2 I_0^2 \quad (4+6)$$

(a) Let us consider propagation of electromagnetic wave in plasma with magnetic field $B_0 = B_0 \hat{z}$. We also consider perpendicular propagation $k \perp B_0$ (say $k = k \hat{x}$) and transverse waves ($k \perp E$), however $E_{\parallel} \parallel B_0$. The plasma has a density n_0 and electron temperature T_e . Calculate the dispersion relation for this electromagnetic wave.

(b) The dispersion relation for electromagnetic waves parallel to B_0 is given by

$$\omega^2 - c^2 k^2 = \frac{\omega_p^2}{1 \mp (\omega_c / \omega)}$$

where - sign is for R wave and + for L wave. Calculate the cutoffs and resonances for these waves.

(7+4)