

**Indian Institute of Technology, Delhi**  
**Department of Physics**  
*EPL208 Electrodynamics and Plasmas*  
 Second Semester 2013-2014

Minor-1

Duration: 1 hour

Marks: 20

Date: 08 Feb. 2014

1. (a) Two metal objects are embedded in a weakly conducting material of conductivity ( $\sigma$ ) as shown in the Figure. Show that the resistance  $R$  between them is related to the capacitance  $C$  of the arrangement by:



$$R = \frac{\epsilon_0}{\sigma C}$$

- (b) In a medium characterized by  $\sigma = 0$ ,  $\mu = \mu_0$ ,  $\epsilon_0$  the electric field is

$$\mathbf{E} = 20 \sin(10^8 t - \beta z) \mathbf{a}_y \quad \text{V/m}$$

Calculate  $\beta$  and  $\mathbf{H}$ . (3+4)

2. Calculate the (time averaged) energy density of an electromagnetic plane wave in a conducting medium. Show that the magnetic contribution always dominates. Also calculate the intensity. Given the fields

$$\mathbf{E}(z,t) = E_0 e^{-k_I z} \cos(k_R z - \omega t + \delta_E) \hat{\mathbf{x}}$$

$$\mathbf{B}(z,t) = B_0 e^{-k_I z} \cos(k_R z - \omega t + \delta_E + \phi) \hat{\mathbf{y}}$$

where,  $k = k_R + i k_I$  is the propagation constant,  $k_{R(I)} = \omega \sqrt{\frac{\epsilon \mu}{2} \left[ 1 + \left( \frac{\sigma}{\epsilon \omega} \right)^2 \right]^{1/2} \pm 1}$  (5)

3. (i) The typical distance between two electrons in a plasma is of the order of  $(n_e)^{-1/3}$ . Show that the potential energy associated with bringing two electrons this close together is much less than their typical kinetic energy, so long as  $n_e \lambda_D^3 \gg 1$ .

- (ii) Two semi-infinite conducting plates form a wedge region between  $\phi = \phi_1$  and  $\phi = \phi_2$  where  $\phi_2 > \phi_1$ . In the region  $\phi_1 < \phi < \phi_2$ ,  $\rho = 0$ ,  $\epsilon = \epsilon_0$ ,  $V = V_0$  at  $\phi = \phi_1$  and  $V = 0$  at  $\phi = \phi_2$ . Find  $V$ ,  $\mathbf{E}$  in the region and charge density on the two conducting plates. (4+4)

**Constants:**  $m_e = 9.11 \times 10^{-31}$  Kg  $\epsilon_0 = 8.85 \times 10^{-12}$  F/m

$e = 1.60 \times 10^{-19}$  C

$\mu_0 = 4 \pi \times 10^{-7}$  H/m

$$\nabla^2 t = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial t}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial t}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}$$

$$\nabla^2 t = \frac{1}{s} \frac{\partial}{\partial s} \left( s \frac{\partial t}{\partial s} \right) + \frac{1}{s^2} \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}$$

$\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial t}{\partial r}) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial t}{\partial \theta}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 t}{\partial \phi^2}$