23 – 11 – 2016

Department of Physics, I.I.T., Delhi

PYL203: *Major Examination*

All questions are compulsory

Time: 2 hours
Actual Marks: 40

1. (a) Consider the function

$$F = \frac{q^2}{2} tanQ,$$

where q and Q are the old and the new canonical coordinates, respectively. Use the general formula to find the new canonical variables $\{Q, P\}$ in terms of the old canonical variables $\{q, p\}$. Is the transformation canonical? Justify your answer. **(8)**

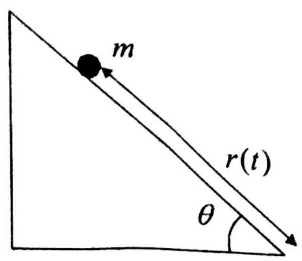
(b) Consider the canonical transformation $\{Q = 2q - p, P = q\}$, where $\{q, p\}$ and $\{Q.P\}$ are the old and the new canonical variables, respectively. Find the corresponding generating function $F_1(q, Q)$. **(4)**

2. A spaceship moves away from Earth with speed v and fires a shuttle craft in the forward direction at a speed v *relative to the spaceship*. Now the pilot of the shuttle craft launches a probe apparatus in the forward direction at speed v *relative to the shuttle craft*. If $v = c/3$, where c is the speed of light in free space, (a) Determine the speed of the *shuttle craft relative to the Earth*, (b) Determine the speed of the *probe relative to the Earth*, (c) A rod of length $L_0 = \sqrt{2}m$ is lying at rest in the probe apparatus along the axis parallel to the direction of motion. An observer in the Earth's system measures it's length. What value will he obtain? **(12)**

3. Determine the principal axes and the principal moments of inertia of a uniform solid ***hemisphere*** of radius b and mass M about its *center of mass*. Assume that the flat surface of the hemisphere lies in the xy plane. (*Hint*: The center of mass does not coincide with the center of the flat surface.) **(14)**

P.T.O.

4. A particle of mass m rests on a flat and smooth plane. The plane is raised to an inclination θ at a constant rate α, when the particle starts to move down the plane. Derive the equations of motion for the particle in terms of $r(t)$ from the Euler-Lagrange equations.



The origin has been put at the bottom of the incline and $r(t)$ is the distance of the particle from the origin. Solve the equations of motion with the initial conditions, $r(0) = r_0, \dot{r}(0) = 0$, and find $r(t)$. **(14)**

5. An oscillatory system, with two degrees of freedom, is characterized by the Lagrangian

$$L = \frac{1}{2}m\dot{x}^2 + m\dot{y}^2 - \frac{1}{2}m\omega_0^2 x^2 - m\omega_0^2 y^2 + 2m\alpha xy,$$

where α is a positive constant. Derive the equations of motion for the column vector

$$\vec{X} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix},$$

in the matrix form using the matrices M and K, as discussed in the tutorial class. Solve the resulting equation and determine the general solutions describing the motion of the system. **(12)**