Department of Physics, I.I.T., Delhi

07 – 10 – 2016

PYL203: *Minor Examination 2*

Full Marks: 40

Actual Marks: 20

All questions are compulsory

1. A uniform distribution of dust in the solar system adds, to the usual force of ***gravitational attraction*** between the planet and the Sun (proportional to $1/r^2$), an additional force $\vec{f}(r) = -mC\vec{r}$, where m is the mass of the planet, C is a constant proportional to the gravitational constant and the density of the dust, and $\vec{r}$ is the radius vector from the sun to the planet (both considered as points). This additional force is *very small* compared to the direct sun-planet gravitational force. Derive the Euler-Lagrange equations. **(a)** Show that the *period of revolution* of the planet on a *circular orbit* of radius r_0 is given by

$$\tau = \tau_0\left(1 - \frac{C\tau_0^2}{8\pi^2}\right),$$

where $\tau_0 = 2\pi r_0^3\sqrt{m/k}$ is the period of circular motion in the absence of the perturbing potential. **(b)** Check, by *linear stability analysis*, whether the circular orbit of the planet (with $r = r_0$), in this combined force field, can be stable or not. **(8+7)**

2. By explicit calculations find the transformation of variables $q \to Q(q,p,t)$ and $p \to P(q,p,t)$ generated by the function

q p

$$F_3(p,Q) = -\left(e^Q - 1\right)^2 tan(p).$$

Using the invariance of Poisson bracket under canonical transformations, show that the resulting transformation is canonical. **(6+9)**

$F_3 - (p,Q)$

$F : F_3(p,Q,t) + \sum_{i=1}^{n} q_i P_i$

$dF = \frac{\partial F}{\partial}$

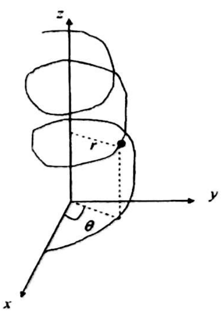

Figure 1: *A particle moving under the influence of gravity along the helix.*

3. A particle of mass m moves under the influence of gravity along the helix $z = k\theta, r = const$, where k is a constant and z is vertical (the geometry is shown in Figure 1 above). Obtain Hamilton's equations of motion. **(10)**

Cos P Cos 2P + Sin P Sin 2P

⇒ Cos² P - Cos P Sin² P + 2 Sin² Cos P

⇒ Cos³ P + Sin² P Cos P

Cos P (s

$\frac{d(\sin p \cos p)}{dp}$

Cos p . Cos p + sin p Sin

Cos 2P

$d\sqrt{1+k\sqrt{x}}$

k .

$1+\sqrt{q}$

$q^{\frac{1}{2}}$

$\frac{1}{2} q^{-1/2}$