Major Test PYL 204

Time : 2 hours. Max. Marks : 40

Q.1. (a) What is Monte-Carlo simulation technique? Give an example to explain it.

(b) Discuss the principle of Power-Residue method for generation of random numbers.

Q.2. Describe the Discrete Fourier Transform (DFT) technique to fit the periodic data $\left[1, \frac{1}{2}, -\frac{1}{2}, -1, -\frac{1}{2}, \frac{1}{2}\right]$ (Period $= 2\pi$)

Q.3. Use the power method to obtain the largest eigenvalue of the matrix $\begin{bmatrix} 58 & 14 & 10 \\ 14 & 60 & 20 \\ 10 & 20 & 40 \end{bmatrix}$ with starting vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. Do five iterations.

Q.4. Give a nine node treatment using a square grid under finite difference method for solving Laplace equation in two dimension given as

$\nabla^2 u = 0$, $0 < x < a$, $0 < y < b$, $a = b = 1$ unit

$u(0,y) = 0$, $u(x,0) = 0$, $u(a,y) = 100 \sin \pi y$, $u(x,b) = 0$

Q.5. Describe Gauss quadrature formula for numerical integration.

Q.6. Two first order reactions (such as those that occur in radioactive decay) may be modeled as $x_1 \xrightarrow{K_1} x_2 \xrightarrow{K_2} x_3$, in which the x_i values represent concentrations of three species and the K_j values are the reaction rates. A prototypical system of equations for such reactions is

$$\frac{dx_1}{dt} = -K_1 x_1 \; ; \quad \frac{dx_2}{dt} = K_1 x_1 - K_2 x_2 \; ; \quad \frac{dx_3}{dt} = K_2 x_2$$

Apply Runge-Kutta method for $x_1 = 1$, $x_2 = 0$, $x_3 = 0$ at $t = 0$, $K_1 = 3$, $K_2 = 1$ to end when x_1 reaches a value 0.5.

Q.7. Use Newton-Raphson method to solve $f(v) = \left(p + \frac{a}{v^2}\right)(v - b) - RT = 0$ for the specific volume v of methane gas at a temperature T of 300 K and a pressure p of 5×10^6 Pa. The gas constant for methane is 518 J/(kg.K) and the van der waal's constants have values of 887 Pa. m^6/kg^2 for 'a' and 0.00267 m^3/kg for 'b'. Use the ideal gas law to obtain a starting value v_0 and make three iterations. Calculate upto five decimal places.

Q.8. Given $\phi(x) = \lambda \int_0^{2\pi} \sin x \sin t \, \phi(t) \, dt$

Find $\phi\left(\frac{\pi}{2}\right)$ and $\phi(\pi)$ analytically and numerically.