Indian Institute of Technology Delhi

Department Of Physics

Major Exam: Mathematical Physics (PYL553)

Maximum Marks: 35 | Duration: 2 Hours | Date: 23/11/2017

Q.1 (a) Using the residues method of complex integration evaluate the integral

$$\int_0^{2\pi} \frac{d\theta}{1 - 2k\, cos\theta + k^2},$$

where, k is a real number with $0 < k < 1$.

(b)If $f(t)$ and $g(t)$ are the two piecewise continuous, bounded, and absolutely integrable functions then prove that

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\omega)\, g(\omega)\, e^{-i\omega t} d\omega,$$

where $*$ represents the convolution, and $f(\omega)$ is the Fourier transform of $f(t)$.

(4+4 marks)

Q.2 (a) Using power series method prove that the differential equation $y'' - 2xy' + 2\nu y = 0$, ν a real number, has the solution of the form

$$H_n(x) = \sum_{m=0}^{n/2} \frac{(-1)^m\, n!\, (2x)^{n-2m}}{m!\, (n-2m)!},$$

where n is an even integer.

(b) For the above polynomial, prove the reccurence relations

(i) $H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x)$

(ii) $H_n'(x) = 2xH_n(x) - H_{n+1}(x)$

(iii) $H_n'(x) = 2nH_{n-1}(x)$

(c) Consider the Legendre polynomials

$$P_n(x) = \sum_{m=0}^{K} \frac{(-1)^m (2n-2m)!}{2^n\, m!\, (n-m)!\, (n-2m)!} x^{n-2m}$$

(where $K = n/2$ if n is even; $(n-1)/2$ if n is odd), which are the solutions of the differential equation $(1 - x^2)y'' - 2xy' + n(n+1)y = 0, x \in [-1, 1]$. Construct an orthonormal set of polynomials $\{p_0(x), p_1(x), p_2(x)\}$.

(3+6+5 marks)

Q.3 (a) Transform the equation $(1 - x^2)y'' - xy' + n^2 y = 0, (x \in [-1, 1])$ to SL form and check whether it is a singular SL problem.

(b) Let ψ_1 and ψ_2 are two functions satisfying the boundary conditions of the form

$$\alpha_1 y(a) + \alpha_2 y'(a) = 0;$$

$$\beta_1 y(b) + \beta_2 y'(b) = 0$$

then prove that

$$\int_a^b [L(\psi_1)\psi_2 - \psi_1 L(\psi_2)]dx = 0,$$

where L is the Sturm-Liouville operator.

(c) Using the method of superposition of eigenfunctions find the solution of the differential equation

$$\frac{d^2y}{dx^2} + \frac{1}{4}y = x/2,$$

with the boundary conditions $y(0) = y(\pi) = 0$ associated with it.

(3+5+5 marks)